Math 1210: Worksheet #2 **Fall 2017**

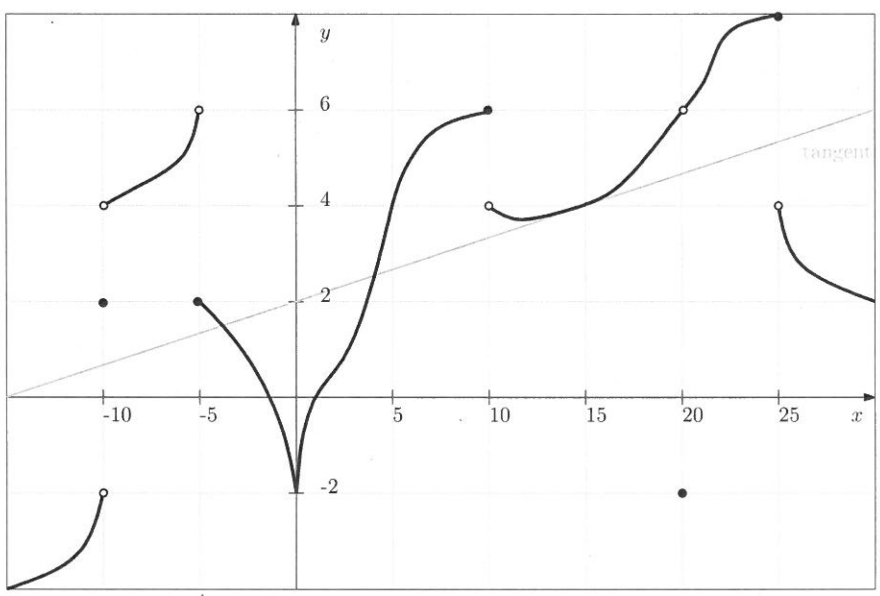
Name: **Solution**	**A#:**	**Section:** **I**

[2] 1. State the formal (ε, δ) definition of $\lim_{x\to a} f(x) = L$. Part marks will be given for an informal definition.

Assuming that $f(x)$ exists for all x in some open interval containing a, except possibly at a. $\lim_{x\to a} f(x) = L$.

if for any number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x-a| < \delta$

[5] 2. Let f be a function whose graph of $y = f(x)$ is given below.



(a) $\lim_{x\to 15} \dfrac{f(x)}{x^2 - 10x} = \dfrac{\lim_{x\to 15} f(x)}{\lim_{x\to 15} x^2 - 10x} = \dfrac{4}{75}$

(b) List all values of a in the interval $(-15, 30)$ for which $\lim_{x\to a^-} f(x) \neq f(a)$: $-10, -5, 20$

(c) List all values of a in the interval $(-15, 30)$ for which limit $\lim_{x\to a} f(x)$ does not exist:

$-10, -5, 10, 25$

(d) The average rate of change of $f(x)$ over the interval $[-5, 10]$ is $\dfrac{6-2}{10-(-5)} = \dfrac{4}{15}$

(e) The instantaneous rate of change of $f(x)$ when $x = 15$ is $\dfrac{2}{15}$

[3] 3. Let $f(x) = \sqrt{x-1}$ and let $a > 1$. Find the instantaneous rate of change of $f(x)$ when $x = a$, as a function (call it g) of a (you are not allowed to use any theory of derivatives).

$$g(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{\sqrt{x-1} - \sqrt{a-1}}{x-a} \times \frac{\sqrt{x-1} + \sqrt{a-1}}{\sqrt{x-1} + \sqrt{a-1}}$$

$$= \lim_{x\to a} \frac{x-1-a+1}{x-a\,(\sqrt{x-1} + \sqrt{a-1})} = \lim_{x\to a} \frac{\cancel{x-a}}{\cancel{(x-a)}(\sqrt{x-1} + \sqrt{a-1})}$$

$$= \lim_{x\to a} \frac{1}{(\sqrt{x-1} + \sqrt{a-1})} = \frac{1}{2\sqrt{a-1}}$$

4. Compute the limit or show that it does not exist.

[3] (a) $\lim_{x\to 1^-} \frac{\sqrt{1+x}-\sqrt{x^2+1}}{x^2-5x+4} \times \frac{\sqrt{1+x}+\sqrt{x^2+1}}{\sqrt{1+x}+\sqrt{x^2+1}} = \lim_{x\to 1^-} \frac{1+x-x^2-1}{(x^2-5x+4)(\sqrt{1+x}+\sqrt{x^2+1})}$

$= \lim_{x\to 1^-} \frac{-x\cancel{(x-1)}}{\cancel{(x-1)}(x-4)(\sqrt{1+x}+\sqrt{x^2+1})} = \frac{-1}{-3(\sqrt{2}+\sqrt{2})} = \frac{1}{6\sqrt{2}} = \frac{\sqrt{2}}{12}$

[3] (b) $\lim_{x\to 1^-} \left(\ln(1-x) - \ln(6-5x-x^2) - \frac{1}{7}\right)$

$= \lim_{x\to 1^-} \ln\left(\frac{1-x}{6-5x-x^2}\right) - 1/7 = \lim_{x\to 1^-} \ln\left(\frac{\cancel{1-x}}{\cancel{(1-x)}(x+6)}\right) - 1/7$

$= \lim_{x\to 1^-} \ln\left(\frac{1}{x+6}\right) - 1/7 = \ln(1/7) - 1/7$

[4] (c) $\lim_{x\to 1} \frac{x^2-1}{|1-x|}$ = Does not exist as $\lim_{x\to 1^+} \frac{x^2-1}{|1-x|} \neq \lim_{x\to 1^-} \frac{x^2-1}{|1-x|}$

$x \geqslant 1 \rightarrow \lim_{x\to 1^+} \frac{x^2-1}{x-1} = \lim_{x\to 1^+} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = 2$

$x < 1 \rightarrow \lim_{x\to 1^-} \frac{x^2-1}{1-x} = \lim_{x\to 1^-} \frac{-\cancel{(1-x)}(x+1)}{\cancel{1-x}} = -2$