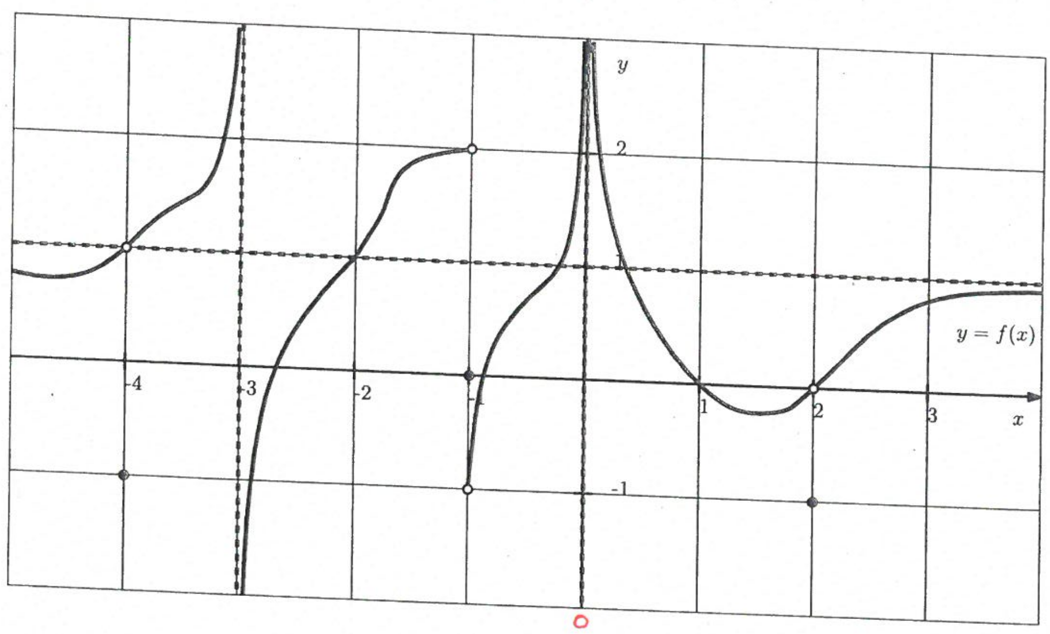
Math 1210: Quiz #3

Fall 2017

Name: Math (solutions)	A#:	Section:

[6] 1. Let f be a function whose graph of $y = f(x)$ is given below.



Then

(a) $\lim_{z \to -4^-} f(z) =$ 1

(b) $\lim_{s \to -3^+} f(s) =$ $-\infty$

(c) $\lim_{z \to -1^-} f(z) =$ 2

(d) List all numbers a for which $\lim_{s \to a} f(s)$ does not exist: $-3, -1, 0$

(e) List all horizontal asymptotes: $y = 1$

(f) List all vertical asymptotes: $x = -3$, $x = 0$

[2] 2. List all vertical asymptotes of $y = \dfrac{(x+2)^3(x-3)^2 \ln|x|}{(x+3)^2(x+2)^2(x-3)^3}$: $x = 3$, $x = 0$, $x = -3$

[redacted]
V3.CD

[4] 3. Find all horizontal asymptotes of $y = \dfrac{e^{2x} - 2e^{-3x}}{3e^{2x} + 5e^{-3x}}$.

As $x \to \infty$ $y \to \dfrac{e^{2x}}{3e^{2x}} \to \dfrac{1}{3}$, hence $y = \frac{1}{3}$ is a horizontal asymptote

As $x \to -\infty$, $y \to \dfrac{-2e^{-3x}}{5e^{-3x}} \to \dfrac{-2}{5}$, hence $y = \frac{-2}{5}$ is also a horizontal asymptote

[8] 4. Compute the limits or show that they do not exist.

(a) $\displaystyle\lim_{t\to\infty} \frac{t^2 + \sin t}{3t^2 - 2\ln(t)}$ $\longrightarrow$ $\displaystyle\lim_{t\to\infty} \frac{t^2}{3t^2} = \frac{1}{3}$, since $\sin t$ is bounded and polynomials grow faster than logs.

(b) $\displaystyle\lim_{x\to\infty} \left(\sqrt{x^2 - 2x + 5} - x\right)$ $\displaystyle = \lim_{x\to\infty} \left(\sqrt{x^2-2x+5} - x\right)\left(\frac{\sqrt{x^2-2x+5}+x}{\sqrt{x^2-2x+5}+x}\right)$

$$= \lim_{x\to\infty} \left(\frac{x^2-2x+5-x^2}{\sqrt{x^2-2x+5}+x}\right)$$

$$= \lim_{x\to\infty} \left(\frac{-2x+5}{\sqrt{x^2-2x+5}+x}\right) = \lim_{x\to\infty} \frac{-2x}{x+x}$$

$$= -1$$

[redacted]

v3. CD

Math 1210: Worksheet #3 **Fall 2017**

Name: Michael (solutions)	A#:	Section:

[2] 1.

(a) List vertical asymptotes of

$$y = \frac{x^2+3x-10}{x^2-4} + \ln|x^2-x|.$$

$$= \frac{(x+5)(x-2)}{(x+2)(x-2)} + \ln|x| + \ln|x-1|$$

vertical

Hence asymptote at $x = 0$ and $x = 1$, and $x = -2$

(b) List horizontal asymptotes of $y = \tan^{-1}(x)$.

Since $\lim_{x\to\infty} \tan^{-1} x = \pi/2$

and $\lim_{x\to-\infty} \tan^{-1} x = -\pi/2$, The horizontal asymptotes of $\tan^{-1}x$ are $\pm\pi/2$

[6] 2. Find horizontal asymptotes of $y = \dfrac{1-3x}{\sqrt{x^2-x+3}}$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{1-3x}{x\sqrt{1-1/x+3/x^2}} = \lim_{x\to\infty} \frac{-3x}{x} = -3$$

$$\lim_{x\to-\infty} y = \lim_{x\to-\infty} \frac{1-3x}{-x\sqrt{1-1/x+3/x^2}} = \lim_{x\to-\infty} \frac{-3x}{-x} = +3$$

Horizontal asymptote at $y = \pm 3$

[12] 3. Compute the limits (as a number, as ∞ , as $-\infty$, or as 'does not exist'; whichever is most precise).

(a) $\lim_{x\to 1^+} \frac{x-2}{x^2+2x-3}$ $= \lim_{x\to 1^+} \frac{(x-2)}{(x-2)(x-1)} = \lim_{x\to 1^+} \frac{1}{x-1} = \boxed{+\infty}$

(b) $\lim_{x\to 0} \frac{2\sin x}{x} + \lim_{t\to -\infty} \frac{\sin t}{2t} + \lim_{s\to \frac{\pi}{6}} \frac{\sin s}{s}$ $= 2\lim_{x\to 0} \frac{\sin x}{x} + \frac{1}{2}\lim_{t\to -\infty} \frac{\sin t}{t} + \frac{\sin(\pi/6)}{\pi/6}$

$= 2(1) + \frac{1}{2}(0) + \frac{6}{\pi}\left(\frac{1}{2}\right) = \boxed{2 + \frac{3}{\pi}}$

Since 0 times bounded is 0.

(c) $\lim_{x\to \infty} \left(e^x - \sqrt{e^{2x} - e^x + 1}\right)$ $= \lim_{x\to\infty} \left(e^x - \sqrt{e^{2x}-e^x+1}\right)\left(\frac{e^x+\sqrt{e^{2x}-e^x+1}}{e^x+\sqrt{e^{2x}-e^x+1}}\right)$

$= \lim_{x\to\infty} \frac{e^{2x} - e^{2x} + e^x - 1}{e^x+\sqrt{e^{2x}-e^x+1}} = \lim_{x\to\infty} \frac{e^x - 1}{e^x+\sqrt{e^{2x}-e^x+1}}$

$= \frac{1}{2}$

(d) $\lim_{x\to -\infty} \left(x^3 - 3x + 2e^{-x}\right)$ $= \lim_{x\to -\infty} 2e^{-x} = \boxed{+\infty}$

Since exponentials grow faster than polynomials.

v3.CD