

Name: SOLUTION

A#:

Section: A, B

[2] 1.

(a) List vertical asymptotes of

$$y = \frac{x^2 + 3x - 10}{x^2 - 4} + \ln|x^2 - x| = \frac{(x-2)(x+5)}{(x-2)(x+2)} + \ln|x(x-1)|$$

$$\text{V.A.} : x = -2, x = 0, x = 1$$

(b) List horizontal asymptotes of  $y = \tan^{-1}(x)$ .

$$\text{H.A.} : y = \frac{\pi}{2}, y = -\frac{\pi}{2}$$

[6] 2. Find horizontal asymptotes of  $y = \frac{1-3x}{\sqrt{x^2-x+3}}$ 

$$\lim_{x \rightarrow \infty} \frac{1-3x}{\sqrt{x^2-x+3}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 3}{\sqrt{1 - \frac{1}{x} + \frac{3}{x^2}}} = \frac{0-3}{\sqrt{1-0+0}} = \boxed{-3}$$

$$\lim_{x \rightarrow -\infty} \frac{1-3x}{\sqrt{x^2-x+3}} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x} - 3}{\sqrt{1 - \frac{1}{x} + \frac{3}{x^2}}} = \boxed{3}$$

$$\text{H.A.} : \boxed{y = -3} \text{ and } \boxed{y = 3}$$

[12]

3. Compute the limits (as a number, as  $\infty$ , as  $-\infty$ , or as 'does not exist'; whichever is most precise).

$$(a) \lim_{x \rightarrow 1^+} \frac{x-2}{x^2+2x-3} = \lim_{x \rightarrow 1^+} \frac{\overset{-1}{x-2}}{\underset{4}{x+3} \underset{0^+}{x-1}} = \lim_{x \rightarrow 1^+} \frac{-1}{4(x-1)} = \boxed{-\infty}$$

$$(b) \lim_{x \rightarrow 0} \frac{2 \sin x}{x} + \lim_{t \rightarrow -\infty} \frac{\sin t}{2t} + \lim_{s \rightarrow \frac{\pi}{6}} \frac{\sin s}{s} = 2| + 0 + \frac{\sin(\frac{\pi}{6})}{\frac{\pi}{6}} = \boxed{2 + \frac{3}{\pi}}$$

$$(c) \lim_{x \rightarrow \infty} (e^x - \sqrt{e^{2x} - e^x + 1}) = \lim_{x \rightarrow \infty} \frac{(e^x - \sqrt{e^x - e^x + 1})(e^x + \sqrt{e^x - e^x + 1})}{e^x + \sqrt{e^x - e^x + 1}}$$

$$= \lim_{x \rightarrow \infty} \frac{e^{2x} - (e^{2x} - e^x + 1)}{e^x + \sqrt{e^x - e^x + 1}} = \lim_{x \rightarrow \infty} \frac{e^x - 1}{e^x + \sqrt{e^{2x} - e^x + 1}} = \lim_{x \rightarrow \infty} \frac{1 - e^{-x}}{1 + \sqrt{1 - e^{-x}}}$$

$$= \frac{1 - 0}{1 + \sqrt{1 - 0}} = \boxed{\frac{1}{2}}$$

$$(d) \lim_{x \rightarrow -\infty} (x^3 - 3x + 2e^{+x}) = \lim_{x \rightarrow -\infty} x^3 \left( 1 - \frac{3}{x^2} + \frac{2e^{+x}}{x^3} \right) = \boxed{-\infty}$$