Name: Solution	A#:	Section: H

[2] 1.

(a) List vertical asymptotes of

$$y = \frac{x^2+4x-12}{x^2-4} + \ln|x-x^2|.$$

$$y = \frac{(x-2)(x+6)}{(x-2)(x+2)} + \ln|x-x^2| = \frac{x+6}{x+2} + \ln|x(1-x)|$$

$x+2=0$, $x=-2$ $\qquad$ $x(1-x)=0$, $x=0$, $x=1$

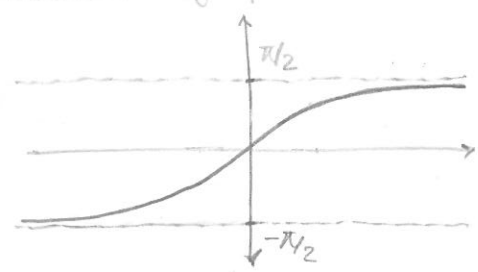
$x=0$, $x=1$, and $x=-2$ are the vertical asymptotes of y.

(b) List horizontal asymptotes of $y = \tan^{-1}(x)$.

$$\lim_{x\to\infty} \tan^{-1}(x) = \pi/2$$

$$\lim_{x\to-\infty} \tan^{-1}(x) = -\pi/2$$

$y=\pi/2$ and $y=-\pi/2$ are the horizontal asymptotes.



[6] 2. Find horizontal asymptotes of $y = \dfrac{1-3x}{\sqrt{x^2-x+3}}$

$$\lim_{x\to\infty} \frac{1-3x}{\sqrt{x^2-x+3}} = \lim_{x\to\infty} \frac{1/x-3}{\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}+\frac{3}{x^2}}} = \lim_{x\to\infty} \frac{\overbrace{1/x}^{\text{approaches } 0}-3}{\sqrt{1-\underbrace{1/x}_{\text{approaches } 0}+\underbrace{3/x^2}_{\text{approaches } 0}}} = \boxed{-3}$$

$$\lim_{x\to-\infty} \frac{1-3x}{\sqrt{x^2-x+3}} = \lim_{x\to-\infty} \frac{\overbrace{\frac{1}{-x}}^{\text{approaches } 0}-\frac{3x}{-x}}{\sqrt{1-\underbrace{1/x}_{\text{approaches } 0}+\underbrace{3/x^2}_{\text{approaches } 0}}} = \boxed{+3}$$

$y=3$, $y=-3$ are the horizontal asymptotes.

[12] 3. Compute the limits (as a number, as ∞ , as $-\infty$, or as 'does not exist'; whichever is most precise).

(a) $\displaystyle \lim_{x\to -2^+} \frac{x+6}{x^2-2x-8}$ $= \displaystyle \lim_{x\to -2^+} \frac{\overbrace{x+6}^{\text{approaches } 4}}{\underbrace{(x+2)}_{\text{positive and approaches } 0}\underbrace{(x-4)}_{\text{approaches } -6}} = \boxed{-\infty}$

(b) $\displaystyle \frac{1}{2}\lim_{x\to 0}\frac{\sin 2x}{x} + \lim_{t\to -\infty}\frac{5\sin t}{t} + \lim_{w\to \frac{\pi}{3}}\frac{\sin w}{2w}$ $= 1 + 5(0) + \dfrac{\frac{\sqrt{3}}{2}}{\frac{2\pi}{3}} = \boxed{1 + \dfrac{3\sqrt{3}}{4\pi}}$

$\displaystyle \lim_{s\to 0}\frac{\sin s}{s} = 1 \implies \frac{1}{2}\lim_{x\to 0}\frac{\sin 2x}{x} = \lim_{x\to 0}\frac{\sin(2x)}{2x} = 1$

$\displaystyle -1 \leq \sin t \leq 1 \implies \lim_{t\to -\infty}\frac{5\sin t}{t} = 0$

(c) $\displaystyle \lim_{x\to\infty}\left(e^x - \sqrt{e^{2x}-e^x+1}\right)$ $\displaystyle \times \frac{e^x + \sqrt{e^{2x}-e^x+1}}{e^x+\sqrt{e^{2x}-e^x+1}} = \lim_{x\to\infty}\frac{e^{2x}-e^{2x}+e^x-1}{e^x+\sqrt{e^{2x}-e^x+1}} \times \frac{1/e^x}{1/e^x}$

$\displaystyle = \lim_{x\to\infty}\frac{1-\overbrace{1/e^x}^{\text{approaches } 0}}{1+\sqrt{1-\underbrace{1/e^x}_{\text{approaches } 0}+\underbrace{1/e^{2x}}_{\text{approaches } 0}}} = \frac{1}{1+\sqrt{1}} = \boxed{1/2}$

(d) $\displaystyle \lim_{x\to -\infty}\left(x^3 - 3x + 2e^{-x}\right)$

we can change the limit to $x\to\infty$ by changing the sign of x, so we get,

$\displaystyle \lim_{x\to\infty}(-x^3+3x+2e^x)$

The exponential grows much faster than x^3, so

$\displaystyle \lim_{x\to\infty}(-x^3+3x+2e^x) \longrightarrow \lim_{x\to\infty} 2e^x = \infty$

$\displaystyle \implies \boxed{\lim_{x\to -\infty}(x^3-3x+2e^{-x}) = \infty}$

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