Math 1210: Worksheet #3 **Fall 2017**

Name: Solution	A#:	Section: I

[2] 1.

(a) List vertical asymptotes of

$$y = \frac{x^2+3x-10}{x^2-4} + \ln|x^2+x|.$$

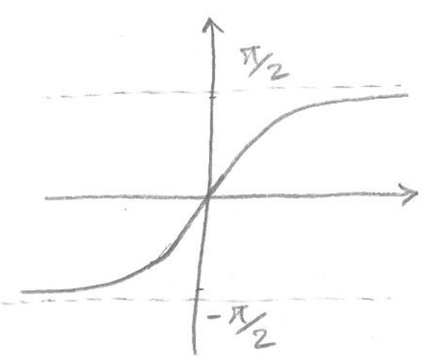
$$y = \frac{(\cancel{x-2})(x+5)}{(\cancel{x-2})(x+2)} + \ln|x^2+x| = \frac{x+5}{x+2} + \ln|x(x+1)|$$

$x+2=0$, $x=-2$

$x(x+1)=0$, $x=0$, $x=-1$

vertical asymptotes : $x=0$, $x=-1$, $x=-2$

(b) List horizontal asymptotes of $y = \tan^{-1}(x)$.



$$\lim_{x\to\infty} \tan^{-1}(x) = \frac{\pi}{2}$$

$$\lim_{x\to-\infty} \tan^{-1}(x) = \frac{-\pi}{2}$$

horizontal asymptotes: $y=\frac{\pi}{2}$ and $y=\frac{-\pi}{2}$

[6] 2. Find horizontal asymptotes of $y = \dfrac{1+x}{\sqrt{x^2-2x+2}}$

$$\lim_{x\to\infty} \frac{1+x}{\sqrt{x^2-2x+2}} = \lim_{x\to\infty} \frac{\frac{(1+x)}{x}}{\sqrt{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{2}{x^2}}} = \lim_{x\to\infty} \frac{\frac{1}{x}+1}{\sqrt{1-\frac{2}{x}+\frac{2}{x^2}}} = 1$$

approaches zero

approaches zero

$$\lim_{x\to-\infty} \frac{1+x}{\sqrt{x^2-2x+2}} = \lim_{x\to-\infty} \frac{\frac{(1+x)}{\sqrt{x^2}}}{\sqrt{\frac{x^2-2x+2}{x^2}}} = \lim_{x\to-\infty} \frac{\frac{1}{-x}+\frac{x}{-x}}{\sqrt{1-\frac{2}{x}+\frac{2}{x^2}}}$$

approaches zero

$$= \lim_{x\to-\infty} \frac{\frac{-1}{x}-1}{\sqrt{1-2/x+2/x^2}} = \frac{-1}{\sqrt{1}} = -1$$

approaches zero

horizontal asymptotes : $y=1$ and $y=-1$

v3.I

[12] 3. Compute the limits (as a number, as ∞ , as $-\infty$, or as 'does not exist'; whichever is most precise).

(a) $\lim_{x\to -1^+} \frac{x+3}{x^2+3x+2}$ $= \lim_{x \to -1^+} \frac{x+3}{(x+2)(x+1)} = \boxed{+\infty}$

approaches 2

approaches 1

approaches 0 and it's positive

(b) $\lim_{x\to 0} \frac{2\sin x}{x} + \lim_{t\to -\infty} \frac{\sin t}{2t} + \lim_{s\to \frac{\pi}{6}} \frac{\sin s}{s}$ $= 2 + 0 + \frac{\frac{1}{2}}{\frac{\pi}{6}} = \boxed{2 + \frac{3}{\pi}}$

$\lim_{x \to 0} \frac{2\sin x}{x} = 2 \lim_{x\to 0} \frac{\sin x}{x} = 2 \times 1 = 2$

$-1 \leq \sin(t) \leq 1 \Rightarrow \lim_{t \to -\infty} \frac{\sin(t)}{2t} = 0$

(c) $\lim_{x\to\infty} \left(e^x - \sqrt{e^{2x}+2e^x+2}\right)$ $\times \frac{e^x + \sqrt{e^{2x}+2e^x+2}}{e^x + \sqrt{e^{2x}+2e^x+2}}$

$= \lim_{x\to\infty} \frac{e^{2x} - (e^{2x}+2e^x+2)}{e^x + e^x\sqrt{1+\frac{2}{e^x}+\frac{2}{e^{2x}}}} = \lim_{x\to\infty} \frac{\cancel{e^x}(-2-\frac{2}{e^x})}{\cancel{e^x}(1+\sqrt{1+\frac{2}{e^x}+\frac{2}{e^{2x}}})}$

$= \lim_{x\to\infty} \frac{-2-\frac{2}{e^x}}{1+\sqrt{1+\frac{2}{e^x}+\frac{2}{e^{2x}}}} = \frac{-2}{1+\sqrt{1}} = \frac{-2}{2} = -1$

approaches zero

approaches zero

(d) $\lim_{x\to-\infty} \left(3x^3 - 3x + 3e^{-x}\right)$ $= \lim_{x\to\infty} (3x^{-3} + 3x + 3e^{x})$

$x < 0$ $\quad x > 0$

The exponential grows faster than polynomial

$\lim_{x\to\infty} 3e^x = \infty$

$\Rightarrow \lim_{x\to -\infty} (3x^3 - 3x + 3e^{-x}) = \infty$