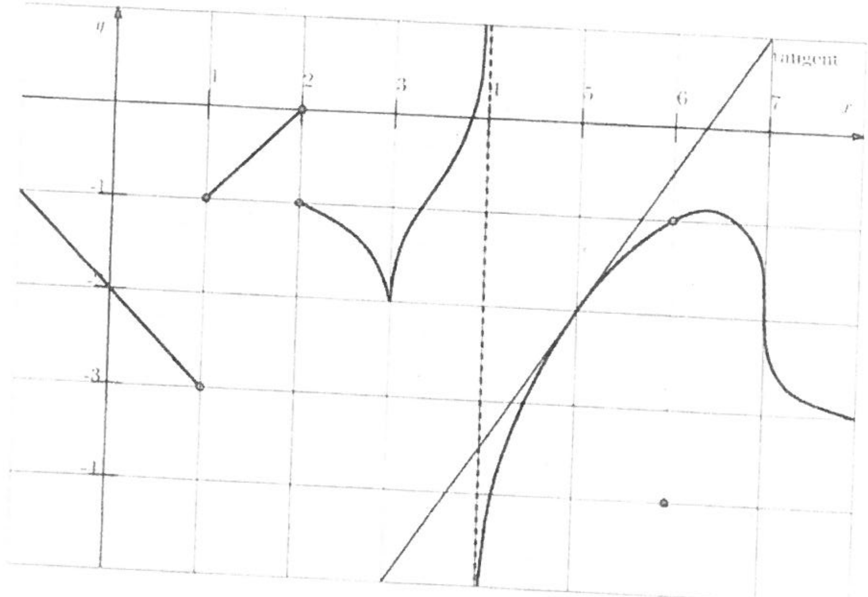
Math 1210: Worksheet #4

Fall 2017

Name: SOLUTION	A#:	Section: A,B

[8] 1. Let f be a function whose graph of $y = f(x)$ is given below.



Fill in the following.

(a) List all x where f is not continuous: 1, 2, 4, 6

(b) List all x where f is left-continuous, but f is not right-continuous: 2

(c) List all x where f is continuous, but not differentiable: 3, 7

(d) $\lim_{t\to 4^+} e^{f(t)} = \lim_{y\to -\infty} e^y = \boxed{0}$

(e) If $g(x) = \dfrac{f(2x+1)}{x^2+1}$, then $g'(x) =$ 2f'([redacted] 2x+1)(x²+1) − 2x f([redacted] 2x+1) / (x²+1)² and $g'(2) = \dfrac{23}{25}$

(f) If $h(x) = e^x f(x)$, then $h'(x) = e^x f(x) + e^x f'(x)$ and the equation of the tangent line to the curve $y = h(x)$ at $x = 5$ is $y + 2e^5 = -\frac{1}{2}e^5(x-5)$

$h(5) = e^5 f(5) = e^5(-2) = -2e^5$

$h'(5) = e^5 f(5) + e^5 f'(5) = e^5(-2) + e^5\left(\frac{3}{2}\right) = -\frac{1}{2}e^5$

$g'(2) = \dfrac{2f'(5)\cdot 5 - 2(2)f(5)}{5^2} = \dfrac{2\cdot\frac{3}{2}\cdot 5 - 2\cdot 2\cdot(-2)}{25} = \dfrac{23}{25}$

[2] 2. Find a function f and a number a such that

$$f'(a) = \lim_{h\to 0} \frac{(3+h)^3 + \tan^{-1}(3(3+h)+2) - 27 - \tan^{-1}(11)}{h}$$

$a =$ 3

$f(x) = x^3 + \tan^{-1}(3x+2)$

[6] 3. Find values of a and b such that the function

$$f(x) = \begin{cases} \tan^{-1}\left(\frac{1}{1-x}\right) & , \text{ for } x < 1 \\ a & , \text{ for } x = 1 \\ b\sin^{-1}\left(\frac{x-1}{x^2-1}\right) & , \text{ for } x > 1 \end{cases}$$

is continuous ~~(everywhere)~~ everywhere.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \tan^{-1}\left(\frac{1}{1-x}\right) = \lim_{y\to\infty} \tan^{-1}(y) = \frac{\pi}{2}.$$

$$f(1) = a.$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} b\sin^{-1}\left(\frac{\cancel{x-1}}{(\cancel{x-1})(x+1)}\right) = b\sin^{-1}\left(\frac{1}{1+1}\right) = b\sin^{-1}\left(\frac{1}{2}\right) = b\cdot\frac{\pi}{6}.$$

For function to be continuous at 1 we need $a = \frac{\pi}{2} = b\cdot\frac{\pi}{6}$, this happens if and only if $\boxed{a = \frac{\pi}{2} \text{ and } b = 3}$

[4] 4. Compute the derivative. **Do not simplify.**

$$\frac{d}{dt}\left(\frac{\sqrt[3]{t}\cos(t)}{1+\sec(2t+1)}\right) =$$

$$= \frac{\left(\frac{1}{3}t^{-2/3}\cos t - \sqrt[3]{t}\sin(t)\right)\left(1+\sec(2t+1)\right) - \sqrt[3]{t}\cos t\cdot 2\cdot\sec(2t+1)\tan(2t+1)}{\left(1+\sec(2t+1)\right)^2}$$

v4.AB