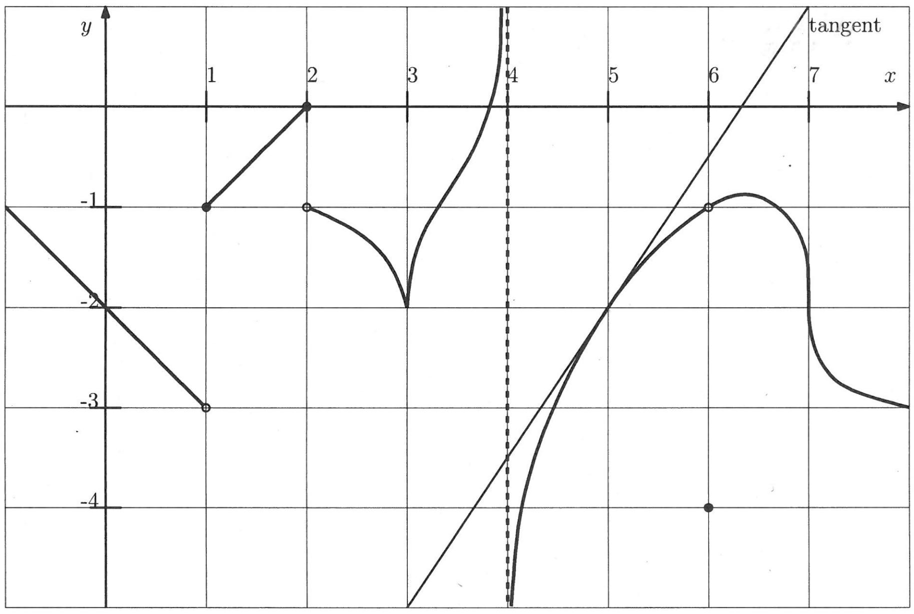
Math 1210: Worksheet #4 **Fall 2017**

Name: Solutions	A#:	Section: 6

[8] 1. Let f be a function whose graph of $y = f(x)$ is given below.



Fill in the following.

(a) List all x where f is not continuous: $x = 1, 2, 4, 6$

(b) List all x where f is left-continuous, but f is not right-continuous: $x = 2$

(c) List all x where f is continuous, but not differentiable: $x = 3, 7$

(d) $\lim_{t\to 4^+} e^{f(t)} = \boxed{0}$ (since $\lim_{t\to 4^+} f(t) = -\infty$)

(e) If $g(x) = \dfrac{f(3x-1)}{x^2+1}$, then $g'(x) = \dfrac{f'(3x-1)\cdot 3\cdot(x^2+1) - f(3x-1)\cdot 2x}{(x^2+1)^2}$ and $g'(2) = \dfrac{3/2\cdot 3\cdot 5 - (-2)\cdot 4}{25} = \boxed{\dfrac{61}{50}}$

(f) If $h(x) = e^x f(x)$, then $h'(x) = e^x f(x) + e^x f'(x)$ and the equation of the tangent line to the curve $y = h(x)$ at $x = 5$ is $(y + 2e^5) = -\frac{e^5}{2}(x-5)$

$m = e^5(f(5) + f'(5)) = e^5(-2 + 3/2)$
$= e^5(-1/2) = -e^5/2$

$x = 5 \Rightarrow y = -2e^5$

$y = -\frac{e^5}{2}x + \frac{5e^5}{2} - 2e^5$

$y = -\frac{e^5}{2}x + \frac{e^5}{2}$

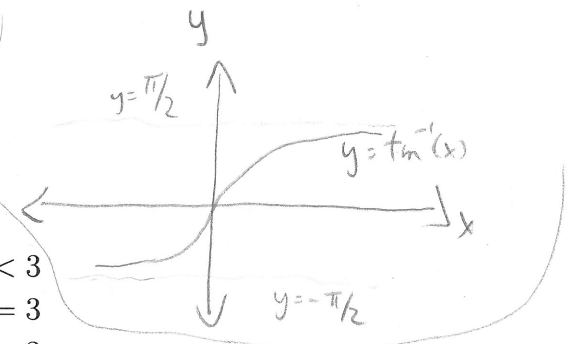
[2] 2. Find a function f and a number a such that

$$f'(a) = \lim_{h\to 0} \frac{e^{(3+h)} + \sin^{-1}(2(3+h)^2 + (3+h) + 1) - e^3 - \sin^{-1}(22)}{h}$$

$a =$ 3

$f(x) =$ $e^x + \sin^{-1}(2x^2 + x + 1)$

[6] 3. Find values of a and b such that the function



$$f(x) = \begin{cases} \tan^{-1}\left(\frac{1}{x-3}\right) & , \text{ for } x < 3 \\ -2a & , \text{ for } x = 3 \\ \frac{1}{b}\cos^{-1}\left(\frac{9-3x}{9-x^2}\right) & , \text{ for } x > 3 \end{cases}$$

is continuous everywhere.

$$\lim_{x \to 3^-} \frac{1}{x-3} = -\infty$$

Need to have: $-2a = \lim_{x\to 3^-} \tan^{-1}\left(\frac{1}{x-3}\right)$

$= -\pi/2$

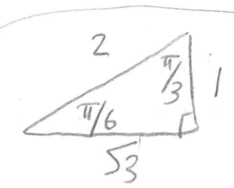
$\therefore 2a = \pi/2 \Rightarrow \boxed{a = \pi/4}$

If $x \neq 3$

$$\frac{9-3x}{(9-x^2)} = \frac{3(3-x)}{(3-x)(3+x)} = \frac{3}{3+x}$$

Need to have: $-\pi/2 = 1/b \lim_{x\to 3^+} \cos^{-1}\left(\frac{9-3x}{9-x^2}\right)$ $\to 1/2$

$= 1/b \cdot \pi/3$



$\cos^{-1}(1/2) = \pi/3$

So $b = \pi/3 \cdot (-2/\pi)$

$\boxed{b = -2/3}$

[4] 4. Compute the derivative. **Do not simplify.**

$$\frac{d}{dt}\left(\frac{\sqrt[3]{t}\cos(t)}{1+\sec(2t+1)}\right)$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{(t^{1/3}\cos(t))'(1+\sec(2t+1)) - t^{1/3}\cos(t)(1+\sec(2t+1))'}{(1+\sec(2t+1))^2}$$

$$= \frac{\left(\frac{1}{3}t^{-2/3}\cos(t) - t^{1/3}\sin(t)\right)(1+\sec(2t+1)) - t^{1/3}\cos(t)\cdot 2\sec(2t+1)\tan(2t+1)}{(1+\sec(2t+1))^2}$$

$(t^{1/3}\cos(t))'$
$= 1/3\, t^{-2/3}\cos(t) + t^{1/3}(-\sin(t))$
$= 1/3\, t^{-2/3}\cos(t) - t^{1/3}\sin(t)$

$(1+\sec(2t+1))'$
$= 0 + \sec(2t+1)\tan(2t+1) \cdot 2$
$= 2\sec(2t+1)\tan(2t+1)$

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