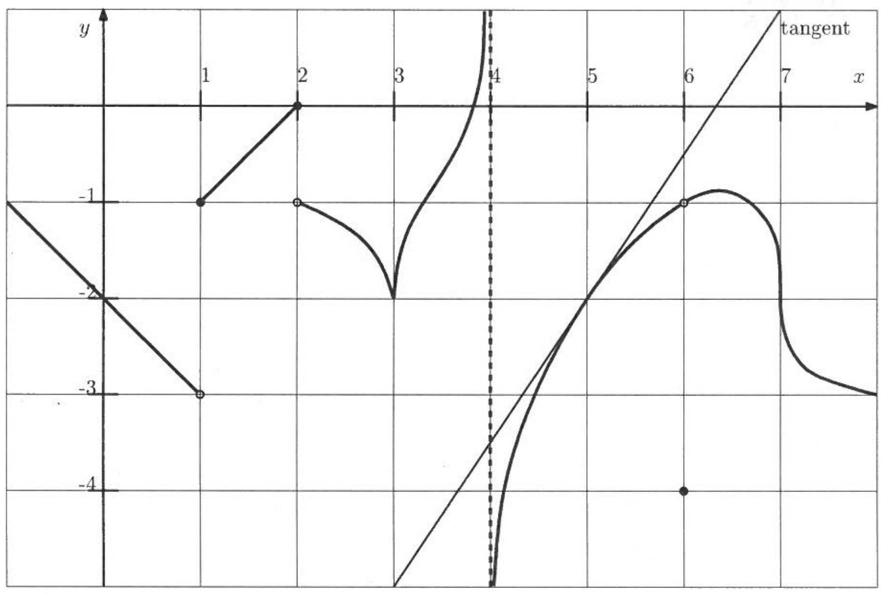
Math 1210: Worksheet #4 **Fall 2017**

Name:	A#:	Section:

[8] 1. Let f be a function whose graph of $y = f(x)$ is given below.



Fill in the following.

(a) List all x where f is not continuous: $1, 2, 4, 6$

(b) List all x where f is left-continuous, but f is not right-continuous: 2

(c) List all x where f is continuous, but not differentiable: $3, 7$

(d) $\lim_{x \to 4^+} e^{f(x)} =$ 0 $\qquad \lim_{x \to 4^+} f(x) = -\infty$

(e) If $g(x) = \dfrac{f(2x+1)}{2x^2+2}$, then $g'(x) = \dfrac{(2x^2+2)2f'(2x+1) - 4xf(2x+1)}{(2x^2+2)^2}$ and $g'(2) = \dfrac{46}{100}$

(f) If $h(x) = e^{2x+1} f(x)$, then $h'(x) = 2e^{2x+1}f(x) + e^{2x+1}f'(x)$ and the equation of the tangent line to the curve $y = h(x)$ at $x = 5$ is $y = -\frac{5}{2}e^{11}x + \frac{21}{2}e^{11}$

$h'(5) = 2e^{11}f(5) + e^{11}f'(5) = -4e^{11} + \frac{3}{2}e^{11} = -\frac{5}{2}e^{11}$

$h(5) = e^{11}f(5) = -2e^{11}$

[2] 2. Find a function f and a number a such that

$$f'(a) = \lim_{h \to 0} \frac{(h-1)^2 + \ln(3(h-1)+4) - 1}{h}$$

$a =$ -1

$f(x) =$ $x^2 + \ln(3x+4)$

[6] 3. Find values of a and b such that the function

$$f(x) = \begin{cases} \cos^{-1}\left(\frac{1}{x-1}\right) & , \text{ for } x < 3 \\ a & , \text{ for } x = 3 \\ b\tan^{-1}\left(\frac{x+1}{3+2x-x^2}\right) & , \text{ for } x > 3 \end{cases}$$

is continuous everywhere.

$$\lim_{x\to 3^-} \cos^{-1}\left(\frac{1}{x-1}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$a = \frac{\pi}{3}$$

$$\lim_{x\to 3^+} b\tan^{-1} \frac{x+1}{3+2x-x^2} = b \lim_{x\to 3^+} \tan^{-1}\left(\frac{\cancel{x+1}}{(\cancel{x+1})(3-x)}\right) = b \lim_{x\to 3^+} \tan^{-1}\frac{1}{3-x} = \frac{-\pi}{2}b$$

$$-\frac{\pi}{2}b = \frac{\pi}{3} \Longrightarrow b = \frac{-2}{3}$$

[4] 4. Compute the derivative. **Do not simplify.**

$$\frac{d}{dt}\left(\frac{\sqrt[4]{t}\sec(t)}{1+\tan(3t+1)}\right)$$

$$= \frac{(1+\tan(3t+1))\frac{d}{dt}\left(\sqrt[4]{t}\sec(t)\right) - \left(\sqrt[4]{t}\sec(t)\right)\frac{d}{dt}\left(1+\tan(3t+1)\right)}{\left(1+\tan(3t+1)\right)^2}$$

$$= \frac{(1+\tan(3t+1))\left[\sec(t)\frac{d}{dt}\left(\sqrt[4]{t}\right) + \sqrt[4]{t}\frac{d}{dt}(\sec(t))\right] - \left(\sqrt[4]{t}\sec(t)\right)\left(3\sec^2(3t+1)\right)}{\left(1+\tan(3t+1)\right)^2}$$

$$= \frac{(1+\tan(3t+1))\left[\sec(t)\frac{t^{-3/4}}{4} + \sqrt[4]{t}\left(\sec(t)\tan(t)\right)\right] - \left(\sqrt[4]{t}\sec(t)\right)\left(3\sec^2(3t+1)\right)}{\left(1+\tan(3t+1)\right)^2}$$