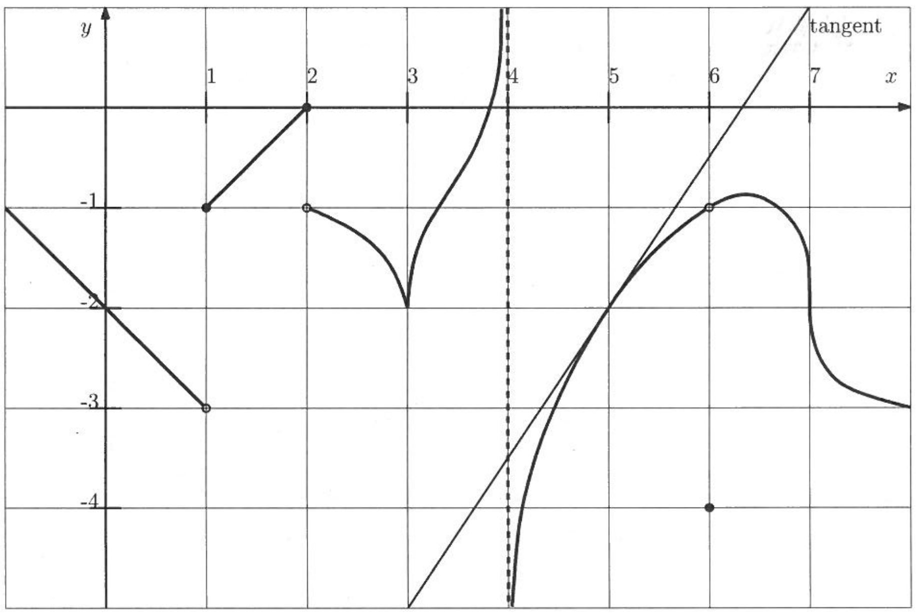
Math 1210: Worksheet #4 **Fall 2017**

Name: Solution	**A#:**	**Section:** H

[8] 1. Let f be a function whose graph of $y = f(x)$ is given below.



Fill in the following.

(a) List all x where f is not continuous: $x = 1, 2, 4, 6$

(b) List all x where f is left-continuous, but f is not right-continuous: $x = 2$

(c) List all x where f is continuous, but not differentiable: $x = 3,\ x = 7$

(d) $\lim\limits_{t \to 4^+} e^{f(t)} =$ 0

$\lim\limits_{t \to 4^+} f(t) = -\infty$

(e) If $g(x) = \dfrac{f(3x-1)}{x^2+1}$, then $g'(x) = \dfrac{3f'(3x-1)(x^2+1) - f(3x-1)(2x)}{(x^2+1)^2}$ and $g'(2) = \dfrac{3f'(5)\cdot 5 - f(5)\cdot 4}{5^2} = \dfrac{61}{50}$

(f) If $h(x) = e^x f(x)$, then $h'(x) =$ $e^x f(x) + f'(x)e^x$ and the equation of the tangent line to the curve $y = h(x)$ at $x = 5$ is $\boxed{y + 2e^5 = -\frac{e^5}{2}(x-5)}$

The slope at $x=5$ is $h'(5) = e^5(-2 + 3/2) = \dfrac{-e^5}{2}$

$x = 5 \rightarrow h(5) = -2e^5$

$y = \dfrac{-e^5}{2}x + \dfrac{5e^5}{2} - 2e^5$

$\boxed{y = \dfrac{-e^5}{2}x + \dfrac{e^5}{2}}$

[2] 2. Find a function f and a number a such that

$$f'(a) = \lim_{h \to 0} \frac{e^{(3+h)} + \sin^{-1}(2(3+h)^2 + (3+h) + 1) - e^3 - \sin^{-1}(22)}{h}$$

$a =$ 3

$f(x) =$ $e^x + \sin^{-1}(2x^2 + x + 1)$

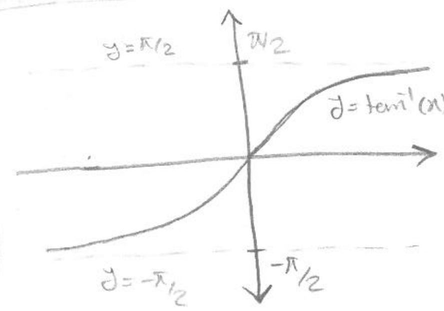
[6] 3. Find values of a and b such that the function

$$f(x) = \begin{cases} \tan^{-1}\left(\frac{1}{x-3}\right) & , \text{ for } x < 3 \\ -2a & , \text{ for } x = 3 \\ \frac{1}{b}\cos^{-1}\left(\frac{9-3x}{9-x^2}\right) & , \text{ for } x > 3 \end{cases}$$

is continuous everywhere.

For the function to be continuous everywhere, it needs to be continuous at $x=3$. Therefore,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$



$$\lim_{x \to 3^-} \frac{1}{x-3} = -\infty$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \tan^{-1}\left(\frac{1}{x-3}\right) = -\pi/2$$

$$\lim_{x \to 3^-} f(x) = f(3) \Rightarrow \pi/2 = -2a \Rightarrow \boxed{a = \pi/4}$$

if $x \neq 3$

$$\frac{9-3x}{9-x^2} = \frac{3(3-x)}{(3-x)(3+x)} = \frac{3}{3+x}$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{1}{b}\cos^{-1}\left(\underbrace{\frac{9-3x}{9-x^2}}_{\to 1/2}\right) = \frac{\pi}{2b}$$

$$\cos(\pi/3) = 1/2$$

$$\Rightarrow \boxed{\cos^{-1}(1/2) = \pi/3}$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) \Rightarrow -\frac{\pi}{2} = \frac{\pi}{3b} \Rightarrow \boxed{b = \frac{-2}{3}}$$

[4] 4. Compute the derivative. **Do not simplify.**

$$\frac{d}{dt}\left(\frac{\sqrt[3]{t}\cos(t)}{1+\sec(2t+1)}\right) = \frac{d}{dt}\left(\frac{t^{1/3}\cos(t)}{1+\sec(2t+1)}\right)$$

$$= \frac{\left(\frac{1}{3}t^{-2/3}\cos(t) - t^{1/3}\sin(t)\right)\left(1+\sec(2t+1)\right) - \left(t^{1/3}\cos(t)\right)\left(2\sec(2t+1)\tan(2t+1)\right)}{\left(1+\sec(2t+1)\right)^2}$$

$$\frac{d}{dt}\frac{f(t)}{g(t)} = \frac{f'(t)g(t) - f(t)g'(t)}{g^2(t)}$$