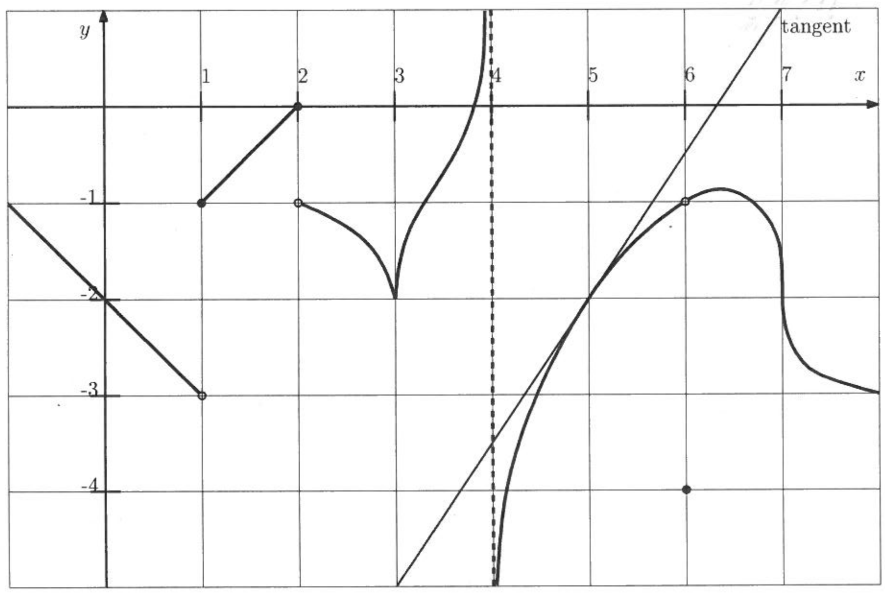
Math 1210: Worksheet #4 **Fall 2017**

Name:	A#:	Section:

[8] 1. Let f be a function whose graph of $y = f(x)$ is given below.



Fill in the following.

(a) List all x where f is not continuous: 1, 2, 4, 6

(b) List all x where f is left-continuous, but f is not right-continuous: 2

(c) List all x where f is continuous, but not differentiable: 3, 7

(d) $\lim_{x\to 4^+} e^{f(x)} =$ 0 $\qquad \lim_{x\to 4^+} f(x) = -\infty$

(e) If $g(x) = \dfrac{f(2x+1)}{2x^2+2}$, then $g'(x) = \dfrac{(2x^2+2)\times 2f'(2x+1) - 4x f(2x+1)}{(2x^2+2)^2}$ and $g'(2) = \dfrac{10\times 2\times \frac{3}{2} - 8\times(-2)}{100} = \dfrac{46}{100}$

(f) If $h(x) = e^{x+1}f(x)$, then $h'(x) = e^{x+1}f(x) + e^{x+1}f'(x)$ and the equation of the tangent line to the curve $y = h(x)$ at $x = 5$ is $y = \frac{-1}{2}e^{6}x + \frac{e^{6}}{2}$

$h'(5) = e^6 f(5) + e^6 f'(5) = -2e^6 + \frac{3}{2}e^6 = \frac{-1}{2}e^6$

$h(5) = e^6 f(5) = -2e^6$

[2] 2. Find a function f and a number a such that

$$f'(a) = \lim_{h\to 0} \frac{(1+h)^2 + \ln(3(1+h)-2) - 1}{h}$$

$a =$ 1

$f(x) = x^2 + \ln(3x-2)$

[6] 3. Find values of a and b such that the function

$$f(x) = \begin{cases} \cos^{-1}\left(\frac{1}{x-1}\right) & , \text{ for } x < 3 \\ a & , \text{ for } x = 3 \\ b\tan^{-1}\left(\frac{x+1}{3+2x-x^2}\right) & , \text{ for } x > 3 \end{cases}$$

is continuous everywhere.

$$\lim_{x \to 3^-} \cos^{-1}\left(\frac{1}{x-1}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$a = \frac{\pi}{3}$$

$$\lim_{x \to 3^+} \tan^{-1}\left(\frac{x+1}{3+2x-x^2}\right) = \lim_{x \to 3^+} \tan^{-1}\left(\frac{\cancel{(x+1)}}{(x+1)(3-x)}\right) = \lim_{x \to 3^+} \tan^{-1}\left(\frac{1}{3-x}\right) = \frac{-\pi}{2}$$

$$\lim_{x \to 3^+} b\tan^{-1}\left(\frac{x+1}{3+2x-x^2}\right) = b \times \frac{-\pi}{2}$$

$$b \times \frac{-\pi}{2} = a = \frac{\pi}{3} \implies b = \frac{-2}{3}$$

[4] 4. Compute the derivative. **Do not simplify.**

$$\frac{d}{dt}\left(\frac{\sqrt[3]{t}\cos(t)}{1+\sec(3t+1)}\right)$$

$$\frac{(1+\sec(3t+1))\frac{d}{dt}\left(\sqrt[3]{t}\cos(t)\right) - \left(\sqrt[3]{t}\cos(t)\right)\frac{d}{dt}\left(1+\sec(3t+1)\right)}{(1+\sec(3t+1))^2}$$

$$= \frac{(1+\sec(3t+1))\left[\cos(t)\frac{d}{dt}(\sqrt[3]{t}) + \sqrt[3]{t}\frac{d}{dt}(\cos(t))\right] - \left(\sqrt[3]{t}\cos(t)\right)\left[3\sec(3t+1)\tan(3t+\right.}{(1+\sec(3t+1))^2}$$

$$\frac{(1+\sec(3t+1))\left[\cos(t)\frac{t^{-2/3}}{3} + \sqrt[3]{t}(-\sin(t))\right] - \left(\sqrt[3]{t}\cos(t)\right)\left(3\sec(3t+1)\tan(3t+1)\right)}{(1+\sec(3t+1))^2}$$