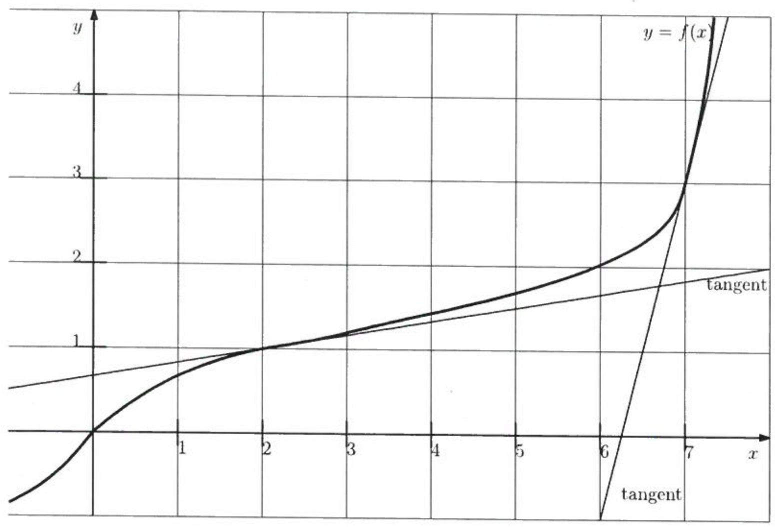
Math 1210: Quiz #5 — Fall 2017

Name: Micah (solutions)	A#:	Section:

[8] 1. Let f be a function whose graph of $y = f(x)$ is given below and let $g = f^{-1}$ be its inverse function.



Fill in the following.

(a) $g(2) =$ 6

(b) $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} =$ $f'(2) = 1/6$

(c) The instantaneous rate of change of $f(x)$ when $x = 7$ is 4

(d) $g'(3) =$ $1/f'(7) = 1/4$

(e) If $h(x) = f(3x^2 + 1)$, then $h'(2) =$ $f'(3(2)^2+1) \cdot (3)(2)(2) = \boxed{f'(13) \cdot 12}$

(f) If $k(x) = g(x)\ln(x)$, then $k'(3) =$ $g'(3)\ln 3 + g(3)/3 = \frac{1}{4}\ln 3 + 7/3$

(g) If $F(x) = \tan^{-1}(f(x))$, then $F'(2) =$ $\left[\frac{1}{1+f(2)^2}\right] f'(2) = \frac{1}{2} \cdot \frac{1}{6} = 1/12$

(h) Tangent line to the curve $y = 2f(x)$ at $x = 2$ is $y = \frac{1}{3}x + \frac{4}{3}$

$m = 2f'(2) = 1/3$

$y = \frac{1}{3}x + b$

$2 = \frac{1}{3}(2) + b$ so $b = 2 - \frac{2}{3} = \frac{4}{3}$

[4] 2. Compute the derivative. **Do not simplify.**

3. $\frac{d}{dt}\left(\sec(e^t) + \tan^{-1}(4) + \sin^{-1}(2t) + \ln(t^4+1)\right)$

$$= \sec(e^t)\tan(e^t)e^t + \frac{1}{\sqrt{1-(2t)^2}} \cdot 2 + \frac{1}{t^4+1} \cdot 4t^3$$

[8] 4. Consider the curve given by $xy^2 = 5 + x^2 + y$. Find the equation of the tangent line given to the curve at the point $(3, -2)$.

Differentiating the curve with respect to x gives:

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(5 + x^2 + y)$$

so $y^2 + 2xy\frac{dy}{dx} = 2x + \frac{dy}{dx}$

at $(3,-2)$, this gives: $4 - 12\frac{dy}{dx} = 6 + \frac{dy}{dx}$ so $13\frac{dy}{dx} = -2$ so $\frac{dy}{dx} = \frac{-2}{13}$

Hence the tangent line is $y = \left(\frac{-2}{13}\right)x + b$ so $b = -2 + \frac{2}{13}(3) = \frac{-20}{13}$

so $\boxed{y = \frac{-2}{13}x - \frac{20}{13}}$ is the tangent line to the curve at the given point

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