

Name: SOLUTION

A#:

Section:

[12] 1. Compute the derivative. Do not simplify.

$$(a) \frac{d}{du} (\sec(e^{\sqrt{u}})) = \sec(e^{\sqrt{u}}) \cdot \tan(e^{\sqrt{u}}) \cdot e^{\frac{1}{2}u} \cdot \frac{1}{2\sqrt{u}}$$

$$(b) \frac{d}{dx} (x \ln(e^x + 1) \tan^{-1}(x^2)) = \ln(e^x + 1) \tan^{-1}(x^2) + x \frac{e^x}{e^x + 1} \tan^{-1}(x^2) + x \ln(e^x + 1) \frac{2x}{1 + x^4} \quad \text{or } (x^2)^2.$$

$$(c) \frac{d}{dx} ((\sin^{-1}(x))^{\sin(x)}) = \frac{d}{dx} e^{\sin x \ln(\sin^{-1} x)} = e^{\sin x \ln(\sin^{-1} x)} \cdot ((\cos x) \ln(\sin^{-1} x) + \sin x \cdot \frac{1}{\sqrt{1-x^2}} \frac{1}{\sin^{-1} x})$$

$$(d) \frac{d}{dt} \frac{\ln(t^4 + 1)}{t^4 + 1} = \frac{\frac{4t^3}{t^4 + 1} (t^4 + 1) - (\ln(t^4 + 1)) 4t^3}{(t^4 + 1)^2}$$

[4]

2. Find the equation of the tangent line to the curve

$$x + y - 1 = x \cos(y)$$

at the point  $(0, 1)$ .

$$1 + y' = \cos y + x(-\sin y) y'$$

$$y'(1 + x \sin y) = \cos y - 1$$

$$y' = \frac{\cos y - 1}{1 + x \sin y}$$

$$y' \Big|_{\substack{x=0 \\ y=1}} = \frac{\cos(1) - 1}{1 + 0 \cdot \sin(1)} = \cos(1) - 1$$

$$\boxed{y - 1 = (\cos(1) - 1)x}$$

[4]

3. Find the equation of the tangent line to the curve

$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point  $(2, 1)$ . (the curve is called lemniscate)

$$6(x^2 + y^2)(2x + 2yy') = 25(2x - 2yy')$$

$$y'(12y(x^2 + y^2) + 50y) = 50x - 12x(x^2 + y^2)$$

$$y' = \frac{50x - 12x(x^2 + y^2)}{12y(x^2 + y^2) + 50y}$$

$$y' \Big|_{\substack{x=2 \\ y=1}} = \frac{50(2) - 12(2)(2^2 + 1^2)}{12(1)(2^2 + 1^2) + 50(1)} = \frac{-20}{110} = -\frac{2}{11}$$

Tangent:

$$\boxed{y - 1 = -\frac{2}{11}(x - 2)}$$