Math 1210: Worksheet #5 **Fall 2017**

Name: [illegible] (solutions)	A#:	Section:

[12] 1. Compute the derivative. **Do not simplify.**

(a) $\frac{d}{du}\left(\sec(e^{\sqrt{u}})\right)$

$= \sec(e^{\sqrt{u}})\tan(e^{\sqrt{u}})\, e^{\sqrt{u}} \cdot \left(\frac{1}{2}u^{-1/2}\right)$

(b) $\frac{d}{dx}\left(x\ln(e^x+1)\tan(x^2)\right)$

$= \ln(e^x+1)\tan(x^2) + \frac{x\tan(x^2)}{e^x+1}\cdot e^x + x\ln(e^x+1)\sec^2(x^2)\cdot 2x$

(c) $\frac{d}{dx}\left((\sin^{-1}(x))^{\sin(x)}\right)$

if $y = \sin^{-1}(x)^{\sin x}$, then $\ln y =$
$= \sin x \ln \sin^{-1} x$

so then $\frac{y'}{y} = \cos x \ln \sin^{-1} x + \sin x\left(\frac{1}{\sin^{-1} x}\cdot\frac{1}{\sqrt{1-x^2}}\right)$

so $y' = (\sin^{-1} x)^{\sin x}\left(\cos x \ln \sin^{-1} x + \sin x\left(\frac{1}{\sin^{-1} x\cdot\sqrt{1-x^2}}\right)\right)$

(d) $\frac{d}{dt}\frac{\ln(t^4+1)}{t^4+1}$ $= \frac{\left(\frac{4t^3}{t^4+1}\right)(t^4+1) - 4t^3\ln(t^4+1)}{(t^4+1)^2}$

[4] 2. Find the equation of the tangent line to the curve

$$x + y - 1 = x\cos(y)$$

at the point $(0, 1)$.

Differentiating gives $\frac{d}{dy}(x + y - 1) = \frac{d}{dy}(x\cos y)$

so $1 + \frac{dy}{dx} = \cos y + \left(-x\sin y \cdot \frac{dy}{dx}\right)$

At $(0,1)$, $\frac{dy}{dx} = -1 + \cos 1$ so $y = (\cos 1 - 1)x + b$, so $b = 1$

Hence the tangent line is $\boxed{y = (\cos 1 - 1)x + 1}$

[4] 3. Find the equation of the tangent line to the curve

$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point $(2, 1)$. (the curve is called lemniscate)

Differentiating with respect to x gives

$\frac{d}{dx}\left(3(x^2+y^2)^2\right) = \frac{d}{dx}\left(25(x^2-y^2)\right)$

so $6(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) = 25\left(2x - 2y\frac{dy}{dx}\right)$

At $(2,1)$, $6(5)\left(4 + 2\frac{dy}{dx}\right) = 25\left(4 - 2\frac{dy}{dx}\right)$

so $120 + 60\frac{dy}{dx} = 100 - 50\frac{dy}{dx}$ so $110\frac{dy}{dx} = -20$

& $\frac{dy}{dx} = -2/11$

Hence $y = \frac{-2}{11}x + b$, $b = 1 + \frac{2}{11}\cdot 2 = \frac{15}{11}$

So the tangent line is

$$\boxed{y = \frac{-2}{11}x + \frac{15}{11}}$$

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