Math 1210: Worksheet #5 **Fall 2017**

Name: Mihai (solutions)	A#:	Section:

[12] 1. Compute the derivative. **Do not simplify.**

(a) $\frac{d}{du}\left(\sec(e^{\sqrt{u}})\right)$

$$= \sec(e^{\sqrt{u}})\tan(e^{\sqrt{u}})\,e^{\sqrt{u}}\left(\tfrac{1}{2}u^{-1/2}\right)$$

(b) $\frac{d}{dx}\left(x\ln(e^x+1)\tan^{-1}(x^2)\right)$

$$= \ln(e^x+1)\tan^{-1}(x^2) + \frac{x\tan^{-1}(x^2)}{e^x+1}\cdot e^x + \frac{x\ln(e^x+1)}{1+(x^2)^2}\cdot 2x$$

(c) $\frac{d}{dx}\left((\sin^{-1}(x))^{\sin(x)}\right)$ if $y = (\sin^{-1}x)^{\sin x}$, then $\ln y = \sin x \ln \sin^{-1} x$

Hence $y'/y = \cos x \ln \sin^{-1} x + \sin x\left(\frac{1}{\sin^{-1}x}\cdot\frac{1}{\sqrt{1-x^2}}\right)$

& thus $\frac{dy}{dx} = (\sin^{-1}x)^{\sin x}\left(\cos x \ln \sin^{-1} x + \frac{\sin x}{\sin^{-1}x\sqrt{1-x^2}}\right)$

(d) $\frac{d}{dt}\frac{\ln(t^4+4)}{t^4+4}$

$$= \frac{\left(\frac{4t^3}{t^4+4}\right)(t^4+4) - 4t^3(\ln(t^4+4))}{(t^4+4)^2}$$

v5.EF

[4] 2. Find the equation of the tangent line to the curve

$$x + y - 1 = x\cos(y)$$

at the point $(0, 1)$.

Differentiating wrt x gives:

$\frac{d}{dx}(x+y-1) = \frac{d}{dx}(x\cos y)$ so $1 + \frac{dy}{dx} = \cos y - x\sin y \cdot \frac{dy}{dx}$

At $(0,1)$, this gives $\frac{dy}{dx} = \cos 1 - 1$. Hence, $y = (\cos 1 - 1)x + b$ so

$b = 1$ so the tangent line is

$$\boxed{y = (\cos 1 - 1)x + 1}$$

[4] 3. Find the equation of the tangent line to the curve

$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point $(2, 1)$. (the curve is called lemniscate)

Differentiating with respect to x gives

$$\frac{d}{dx}\left(3(x^2+y^2)^2\right) = \frac{d}{dx}\left((25)(x^2-y^2)\right)$$

so $6(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) = 25\left(2x - 2y\frac{dy}{dx}\right)$

At $(2,1)$ this gives $6(5)\left(4 + 2\frac{dy}{dx}\right) = 25\left(4 - 2\frac{dy}{dx}\right)$

so $120 + 60\frac{dy}{dx} = 100 - 50\frac{dy}{dx}$ so $110\frac{dy}{dx} = -20$ & $\frac{dy}{dx} = \frac{-2}{11}$

Then $y = mx + b$ so $b = 1 + \frac{2}{11} \cdot 2 = \frac{15}{11}$ hence the tangent line is:

$$\boxed{y = \frac{-2}{11}x + \frac{15}{11}}$$