Math 1210: Worksheet #5 **Fall 2017**

Name: Solution	A#:	Section: H

[12] 1. Compute the derivative. **Do not simplify.**

(a) $\frac{d}{du}\left(e^{\sec(\sqrt[3]{u})}\right) = (\sec(\sqrt[3]{u}))' \cdot e^{\sec(\sqrt[3]{u})}$

$= (\sec(u^{1/3}))' \cdot e^{\sec(u^{1/3})}$

$= \left(\frac{1}{3} u^{-2/3} \cdot \sec(u^{1/3}) \cdot \tan(u^{1/3})\right) \cdot e^{(u^{1/3})}$

(b) $\frac{d}{dx}\left(\ln((e^x+1)^{x+2})\cos(x^2)\right) = \frac{d}{dx}\left((x+2)\ln(e^x+1)\cos(x^2)\right)$

$= (x+2)' \cdot \left(\ln(e^x+1)\cos(x^2)\right) + (x+2)\left(\ln(e^x+1)\cos(x^2)\right)'$

$= 1 \cdot \left(\ln(e^x+1)\cos(x^2)\right) + (x+2)\left(\frac{e^x}{e^x+1} \cdot \cos(x^2) + \ln(e^x+1) \cdot (-\sin(x^2) \cdot 2x)\right)$

$= \ln(e^x+1)\cos(x^2) + (x+2) \cdot \frac{e^x}{e^x+1} \cdot \cos(x^2) - (x+2) \cdot \ln(e^x+1) \cdot \sin(x^2) \cdot 2x$

(c) $\frac{d}{dx}\left((\sin^{-1}(x))^{\sin(x)}\right)$

$f(x) = (\sin^{-1}(x))^{\sin(x)}$

$\ln(f(x)) = \sin(x)\ln(\sin^{-1}(x))$

Implicit Diff:

$\frac{f'(x)}{f(x)} = \cos(x)\ln(\sin^{-1}(x)) + \sin(x) \cdot \frac{\frac{1}{\sqrt{1-x^2}}}{\sin^{-1}(x)}$

$= f'(x)$

$= f(x) \cdot \left(\cos(x)\ln(\sin^{-1}(x)) + \frac{\sin(x)}{\sin^{-1}(x) \cdot \sqrt{1-x^2}}\right)$

$= (\sin^{-1}(x))^{\sin(x)} \cdot \left(\cos(x)\ln(\sin^{-1}(x)) + \frac{\sin(x)}{\sin^{-1}(x)\sqrt{1-x^2}}\right)$

(d) $\frac{d}{dt}\frac{t^6+t^2+3}{\ln(t^6+t^2+3)}$

$= \frac{(t^6+t^2+3)' \cdot \ln(t^6+t^2+3) - (\ln(t^6+t^2+3))' \cdot (t^6+t^2+3)}{(\ln(t^6+t^2+3))^2}$

$= \frac{(6t^5+2t) \cdot \ln(t^6+t^2+3) - \frac{6t^5+2t}{t^6+t^2+3} \cdot (t^6+t^2+3)}{(\ln(t^6+t^2+3))^2}$

1) C) Solution 2:

$$\frac{d}{dx}\left((\sin^{-1}(x))^{\sin(x)}\right)$$

$$= \frac{d}{dx}\left(e^{\sin(x)\ln(\sin^{-1}(x))}\right)$$

$$= e^{\sin(x)\ln(\sin^{-1}(x))} \cdot \left(\sin(x)\ln(\sin^{-1}(x))\right)'$$

$$= e^{\sin(x)\ln(\sin^{-1}(x))} \cdot \left(\cos(x)\ln(\sin^{-1}(x)) + \sin(x) \cdot \frac{1}{\sin^{-1}(x)} \cdot \frac{1}{\sqrt{1-x^2}}\right)$$

$$= e^{\sin(x)\ln(\sin^{-1}(x))} \cdot \left(\cos(x)\ln(\sin^{-1}(x)) + \frac{\sin(x)}{\sin^{-1}(x)\sqrt{1-x^2}}\right)$$

$$= (\sin^{-1}(x))^{\sin(x)} \cdot \left(\cos(x)\ln(\sin^{-1}(x)) + \frac{\sin(x)}{\sin^{-1}(x)\sqrt{1-x^2}}\right)$$

$$e^{\ln f(x)} = f(x)$$

$$\Rightarrow e^{\ln((\sin^{-1}(x))^{\sin(x)})} = (\sin^{-1}(x))^{\sin(x)}$$

$$\Rightarrow e^{\sin(x)\ln(\sin^{-1}(x))} = (\sin^{-1}(x))^{\sin(x)}$$

[4] 2. Find the equation of the tangent line to the curve

$$2x + y - 5 = x\tan^{-1}(y-4)$$

at the point $(0, 5)$.

implicit diff: $2 + y' = \tan^{-1}(y-4) + x \cdot \frac{1}{(y-4)^2+1} \cdot y'$

$$y'\left(1 - \frac{x}{(y-4)^2+1}\right) = \tan^{-1}(y-4) - 2$$

$$\boxed{y' = \frac{\tan^{-1}(y-4) - 2}{1 - \frac{x}{(y-4)^2+1}}}$$

At point $(0,5)$ $\quad m = \frac{\tan^{-1}(1) - 2}{1 - 0} = \pi/4 - 2 = \boxed{\frac{\pi - 8}{4}}$

The equation of tangent line at $(0,5)$:

$$y - 5 = \frac{\pi - 8}{4}(x - 0) \qquad \boxed{y = \frac{\pi-8}{4}x + 5}$$

[4] 3. Find the equation of the tangent line to the curve

$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point $(2, 1)$. (the curve is called lemniscate)

Implicit diff:

$$6(x^2+y^2)\cdot(2x + 2yy') = 25(2x - 2yy')$$

plug in the point and solving it for y', we get:

$$6(4+1)\cdot(4 + 2y') = 25(4 - 2y')$$

$$60(2+y') = 50(2-y')$$

$$6(2+y') = 5(2-y') \Rightarrow 12 + 6y' = 10 - 5y'$$

$$11y' = -2 \Rightarrow \boxed{y' = \frac{-2}{11}}$$

Equation of tangent line at $(2,1)$:

$$y - 1 = \frac{-2}{11}(x - 2)$$

$$y = \frac{-2}{11}x + \frac{4}{11} + 1 \Rightarrow \boxed{y = \frac{-2}{11}x + \frac{15}{11}}$$

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