

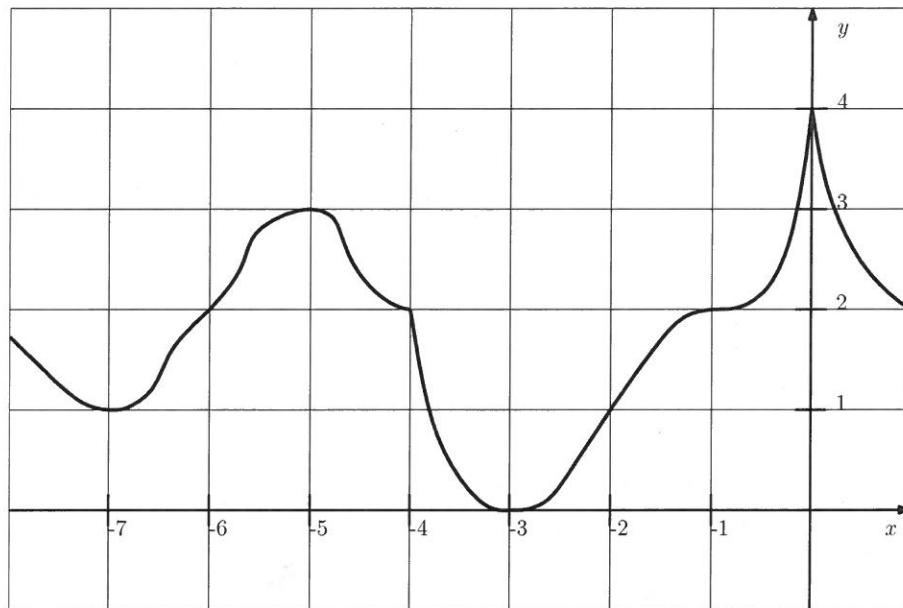
Name: SOLUTION

A#:

Section:

[8]

1. Let  $f$  be a function whose graph of  $y = f(x)$  is given below.



Fill in the following.

(a) The critical ~~values~~<sup>points for</sup> of  $f$  are: -7, -5, -4, -3, -1, 0

(b)  $f$  has local minima at: -7, -3

(c)  $f$  has local maxima at: -5, 0

(d) On the following intervals we have  $f'(x) > 0$ : (-7, -5), (-3, -1), (-1, 0)

(e) The global maximum of  $f$  on  $(-8, 1)$  is 4

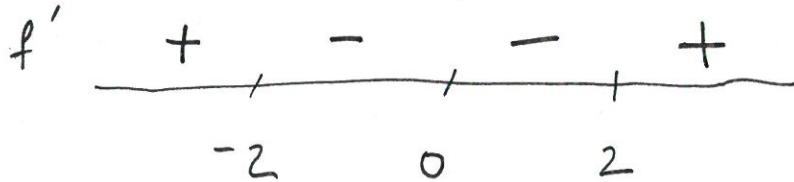
(f) The global minimum of  $f$  on  $(-8, 1)$  is 0

(g) The global maximum of  $f$  on  $[-7, -6]$  is 2

(h) The global minimum of  $f$  on  $[-2, 0]$  is 1

- [3] 2. List and classify critical points of  $f$ , if its **derivative** is given by

$$f'(x) = \frac{(x-2)^3 x^2}{\sqrt[3]{x+2}}$$



Critical points :  $-2$  (local max.),  $0$  (not a local extremum),  $2$  (loc. min)

- [4] 3. Let  $x, y$  be functions of  $t$  related by  $4x^2y^2 = x^4 + y^4$ . Compute  $\frac{dy}{dt}$  in terms of  $x, y, \frac{dx}{dt}$ .

$$8x^2y^2 \frac{dx}{dt} + 8x^2y \frac{dy}{dt} = 4x^3 \frac{dx}{dt} + 4y^3 \frac{dy}{dt}$$

$$\frac{dy}{dt} (8x^2y - 4y^3) = \frac{dx}{dt} (4x^3 - 8x^2y^2)$$

$$\frac{dy}{dt} = \boxed{\frac{dx}{dt} \cdot \frac{4x^3 - 8xy^2}{8x^2y - 4y^3}} = \frac{dx}{dt} \frac{x^3 - 2xy^2}{2x^2y - y^3}$$

- [5] 4. Find the global (absolute) maximum and the global minimum of  $f(x) = x^3 + 6x^2$  on  $[-5, -1]$ .

$$f'(x) = 3x^2 + 12x = 3x(x+4).$$

Critical points :  $-4, 0$   $\leftarrow$  not in the interval

$x$	$f(x)$
-5	25
-4	32
-1	5

$$f(-5) = (-5)^3 + 6(-5)^2 = -125 + 150 = 25$$

$$f(-2) = (-2)^3 + 6(-2)^2 = -8 + 24 = 16$$

$$f(-1) = (-1)^3 + 6(-1)^2 = 5.$$