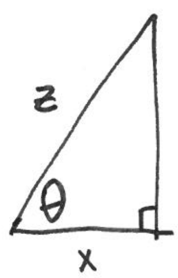
Math 1210: Worksheet #6 **Fall 2017**

Name: SOLUTION	A#:	Section:

[6] 1. Consider a right-angled triangle with catheti (adjacent sides) x and y and hypothenuse (opposite side) z. When $x = 5$ and $y = 12$ we have that x is growing at the rate of $7cm/s$ and the angle θ between x and z is growing at the rate of $\frac{1}{4}rad/s$, (the right angle $\frac{\pi}{2}$ between x and y is fixed throughout the process). What is the rate of change of z at that point?



$$\frac{x}{z} = \cos\theta .$$

$$x = z \cdot \cos\theta$$

$$\frac{dx}{dt} = \frac{dz}{dt}\cos\theta + z(-\sin\theta)\frac{d\theta}{dt}$$

$$\frac{dz}{dt} = \frac{\frac{dx}{dt} + z\sin\theta\frac{d\theta}{dt}}{\cos\theta}$$

When $x=5, y=12$, we have $z = \sqrt{x^2+y^2} = 13$, $\sin\theta = \frac{12}{13}$ and $\cos\theta = \frac{5}{13}$

$$\left.\frac{dz}{dt}\right|_{\substack{x=5\\y=12}} = \frac{7 + 13\cdot\frac{12}{13}\cdot\frac{1}{4}}{\frac{5}{13}} = \frac{10\cdot 13}{5} = \boxed{26}$$

[3] 2. Let x and y be functions of t related by $x^2+y^2 = xy+7$. When $x = 3$ and $y = 2$ we have that $\frac{dx}{dt} = -2$. Find the value of $\frac{dy}{dt}$ at that point.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$$

$$\frac{dy}{dt}(2y-x) = \frac{dx}{dt}(y-2x).$$

$$\frac{dy}{dt} = \frac{dx}{dt}\,\frac{y-2x}{2y-x}.$$

$$\left.\frac{dy}{dt}\right|_{\substack{x=3\\y=2}} = (-2)\,\frac{2-2(3)}{2(2)-3} = \boxed{8}$$

[3] 3. Let f be a function whose **derivative** is

$$f'(x) = \frac{(x+2)^4 \ln(x^2) e^x}{\sqrt[3]{x+9}}.$$

List the intervals where f is increasing and list as well as classify all critical ~~values~~ points.

loc min, neither, loc. max, neither, loc. min.

$$\begin{array}{ccccccccccc} f' & - & & + & & + & & - & & - & & + \\ & & -9 & & -2 & & -1 & & 0 & & 1 & \end{array}$$

this is sufficient for classification

Critical points: -9 (loc. min), -2 (not ext.), -1 (loc. max), 0 (not ext.), 1 (loc. min)

~~Interval~~ Increasing: $(-9,-2), (-2,-1), (1,\infty)$

[8] 4. Find the global maximum and the global minimum of $f(x) = (x^2 - 2x)e^x$ on the interval $[0, 3]$. For **5 bonus marks** find the global maximum and minimum of f on $(-\infty, 0]$ or explain why they do not exist.

$f'(x) = (2x-2)e^x + (x^2-2x)e^x = (x^2-2)e^x.$

Critical points: $-\sqrt{2}$, $\sqrt{2}$

not in the interval.

x	$f(x)$	
0	0	
$\sqrt{2}$	$(2-2\sqrt{2})e^{\sqrt{2}}$	min.
3	$3e^3$	max

Bonus:

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (x^2-2x)e^x = 0$

$f(-\sqrt{2}) = ((-\sqrt{2})^2 - 2(-\sqrt{2}))e^{-\sqrt{2}} = \boxed{(2\sqrt{2}+2)e^{-\sqrt{2}}.\quad \text{max.}}$

$f(-1) = 3e^{-1}.$

$f(-1) < f(-\sqrt{2})$ as f is decreasing on the interval $(-\sqrt{2}, \sqrt{2})$, as $f'(x) < 0$ there.

~~min~~ $\boxed{\text{Min. does not exist}}$