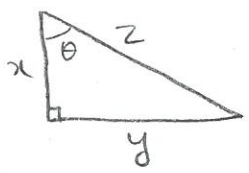
Math 1210: Worksheet #6 Solution **Fall 2017**

Name:	A#:	Section: F

[6] 1. Consider a right-angled triangle with catheti (adjacent sides) x and y and hypothenuse (opposite side) z. When $x = 6$ and $y = 8$ we have that x is growing at the rate of $2cm/s$ and the angle θ between x and z is growing at the rate of $\frac{1}{8}rad/s$, (the right angle $\frac{\pi}{2}$ between x and y is fixed throughout the process). What is the rate of change of z at that point? For **2 bonus marks** determine the rate of change of y at that point.



$$\left.\begin{array}{l} x = 6 \\ y = 8 \end{array}\right\} \longrightarrow \begin{array}{l} x^2 + y^2 = z^2 \\ 36 + 64 = 100 \longrightarrow z = 10 \end{array}$$

$$\cos(\theta) = \frac{x}{z} \longrightarrow \frac{d}{dt}(\cos(\theta)) = \frac{d}{dt}\left(\frac{x}{z}\right)$$

$$\Longrightarrow -\sin(\theta)\frac{d\theta}{dt} = \frac{\frac{dx}{dt}\cdot z - x\frac{dz}{dt}}{z^2}$$

$$\Longrightarrow \frac{-8}{10}\cdot\frac{1}{8} = \frac{2\cdot 10 - 6\cdot\frac{dz}{dt}}{100} \Longrightarrow \frac{dz}{dt} = 5$$

Bonus

$$\sin(\theta) = \frac{y}{z} \longrightarrow \frac{d}{dt}(\sin(\theta)) = \frac{d}{dt}\left(\frac{y}{z}\right)$$

$$\Longrightarrow \cos(\theta)\cdot\frac{d\theta}{dt} = \frac{\frac{dy}{dt}\cdot z - y\cdot\frac{dz}{dt}}{z^2}$$

$$\Longrightarrow \frac{6}{10}\cdot\frac{1}{8} = \frac{\frac{dy}{dt}\cdot 10 - 8\cdot 5}{100} \Longrightarrow \frac{dy}{dt} = \frac{19}{4}$$

[3] 2. Let x and y be functions of t related by $x^2 + y^3 = x^2y + 7$. When $x = 1$ and $y = 2$ we have that $\frac{dx}{dt} = 11$. Find the value of $\frac{dy}{dt}$ at that point.

$$\frac{d}{dt}(x^2 + y^3) = \frac{d}{dt}(x^2y + 7)$$

$$2x\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 2xy\frac{dx}{dt} + x^2\frac{dy}{dt}$$

$$\frac{dy}{dt}(3y^2 - x^2) = 2xy\frac{dx}{dt} - 2x\frac{dx}{dt}$$

$$\frac{dy}{dt}(12 - 1) = 2\cdot 2\cdot 11 - 2\cdot 11 = 22 \longrightarrow \frac{dy}{dt} = 2$$

[3] 3. Let f be a function whose **derivative** is

$$f'(x) = \frac{(x+2)^4 \ln(x^2) e^x}{\sqrt[3]{x+9}}.$$

List the intervals where f is increasing and list as well as classify all critical values.

$f'(x)$	$-$		$+$		$+$		$-$		$-$		$+$
$f(x)$	$\searrow$	-9	$\nearrow$	-2	$\nearrow$	-1	$\searrow$	0	$\searrow$	1	$\nearrow$

$f'(x) = 0 \longrightarrow x = -2, \pm 1$

$f'(x)$ is undefined $\longrightarrow x = 0, -9$

local max at $x = -1$

local min at $x = -9, 1$

no extrema at $x = -2, 0$

$f(x)$ is increasing on $(-9,-2), (-2,-1), (1, \infty)$

[8] 4. Find the global maximum and the global minimum of $f(x) = (x^2 - 2x)e^x$ on the interval $[0, 3]$. For **3 bonus marks** find the global maximum and minimum of f on $(-\infty, 0]$ or explain why they do not exist.

$f'(x) = (2x-2)e^x + (x^2 - 2x)e^x = e^x(2x - 2 + x^2 - \cancel{2x})$

$f'(x) = e^x(x^2 - 2)$

$f'(x) = 0 \longrightarrow x^2 - 2 = 0 \longrightarrow x = \pm\sqrt{2}$

$f(0) = 0$

$f(\sqrt{2}) = (2 - 2\sqrt{2})e^{\sqrt{2}} < 0$

$f(3) = 3e^3 > 0$

abs. max : $f(3) = 3e^3$

abs. min : $f(\sqrt{2}) = (2 - 2\sqrt{2})e^{\sqrt{2}}$

— Bonus —

$f'(x) = (x^2 - 2)e^x$ and from above work we know $x = -\sqrt{2}$ is critical point.

$f'(x)$	$+$		$-$	
$f(x)$	$\nearrow$	$-\sqrt{2}$	$\searrow$	0

So, we have a local max at $x = -\sqrt{2}$

and $f(-\sqrt{2}) = \dfrac{2 + 2\sqrt{2}}{e^{\sqrt{2}}} > 0$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} (x^2 - 2x)e^x = \lim\limits_{x \to \infty} \dfrac{x^2 + 2x}{e^x} = 0 \Longrightarrow$ horizontal asymptote $y = 0$

Therefore, we have global min of 0 at $x = 0$

and global max of $\dfrac{2(1+\sqrt{2})}{e^{\sqrt{2}}}$ at $x = -\sqrt{2}$

v6.EF