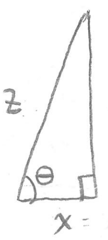
Math 1210: Worksheet #6 — Fall 2017

Name: Solutions	A#:	Section: G

[6] 1. Consider a right-angled triangle with catheti (adjacent sides) x and y and hypothenuse (opposite side) z. When $x = 5$ and $y = 12$ we have that x is growing at the rate of $7cm/s$ and the angle θ between x and z is growing at the rate of $\frac{1}{4}rad/s$, (the right angle $\frac{\pi}{2}$ between x and y is fixed throughout the process). What is the rate of change of z at that point?



$$\left.\begin{aligned} dx/dt &= 7cm/s \\ \frac{d\theta}{dt} &= 1/4\ rad/s \end{aligned}\right\}$$ when $x = 5$, $y = 12$. When $x = 5$, $y = 12$ we have $z^2 = 5^2 + 12^2 \Rightarrow z^2 = 169 \Rightarrow z = 13$

Now $\cos\theta = x/z$ relates x and z. Take the derivative w.r.t. t

$$d/dt(\cos\theta) = d/dt\left(\frac{x}{z}\right) \Rightarrow \boxed{-\sin(\theta)\frac{d\theta}{dt} = \frac{dx/dt\, z - x\, dz/dt}{z^2}}$$

We want to find dz/dt: $-12/13 \cdot 1/4 = \dfrac{7 \cdot 13 - 5 \cdot \frac{dz}{dt}}{169}$

Solving: $-3/13 = 7/13 - \dfrac{5\frac{dz}{dt}}{169}$

$-10/13 = \dfrac{-5}{169} \cdot \dfrac{dz}{dt}$ $\quad 10 \cdot 13 = 5\, dz/dt \Rightarrow \boxed{\frac{dz}{dt} = 2 \cdot 13 = 26\ cm/s}$

[3] 2. Let x and y be functions of t related by $x^2 + y^2 = xy + 7$. When $x = 3$ and $y = 2$ we have that $\frac{dx}{dt} = -2$. Find the value of $\frac{dy}{dt}$ at that point.

First differentiate: $d/dt(x^2 + y^2 = xy + 7) \Rightarrow 2x\, dx/dt + 2y\, dy/dt = y\, dx/dt + x\, dy/dt + 0$

$2y\, dy/dt - x\, dy/dt = y\, dx/dt - 2x\, dx/dt$

$$\Rightarrow \boxed{dy/dt = \frac{(y-2x)\, dx/dt}{(2y-x)}}$$

$$\therefore\ dy/dt = \frac{(2-6)\cdot(-2)}{(4-3)} = \frac{(-4)\cdot(-2)}{1} = 8$$

$$\boxed{dy/dt = 8}$$

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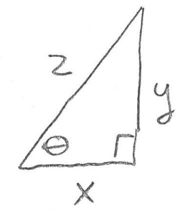
Bonus:

Given: $dx/dt = 7cm/s$

$d\theta/dt = 1/4 \ rad/s$

We calculated: $dy/dt = 26 cm/s$



When $x=5$, $y=12$ we have $z^2 = 25+144 = 619 \Rightarrow z = 13$

One way: $x^2+y^2=z^2 \Rightarrow 2x\, dx/dt + 2y\, dy/dt = 2z\, dz/dt$

$\Rightarrow 2\cdot5\cdot7 + 2\cdot12\cdot dy/dt = 2\cdot13\cdot26$

$\Rightarrow 5\cdot7 + 12\cdot dy/dt = 13\cdot26$

$\Rightarrow dy/dt = 303/12 = 101/4 \ cm/s$

Another way: $\tan\theta = y/x \Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{dy/dt\cdot x - y\, dx/dt}{x^2}$

$\Rightarrow \left(\frac{z}{x}\right)^2 \frac{d\theta}{dt} = \frac{dy/dt\cdot x - y\, dx/dt}{x^2}$

$\Rightarrow \left(\frac{13}{5}\right)^2 \cdot \frac{1}{4} = \frac{5\, dy/dt - 12\cdot 7}{25}$

$5\, dy/dt = \frac{13\cdot13}{25\cdot4}\cdot 25 + 84 \Rightarrow dy/dt = \frac{189+336}{5\cdot4} = \frac{505}{5\cdot4} = \frac{101}{4} cm/s$

Recall $\ln(x^2)$ undefined for $x=0$

Recall $\ln(x^2)=0$ when $x^2=1$

[3] 3. Let f be a function whose **derivative** is

$$f'(x) = \frac{(x+2)^4 \ln(x^2) e^x}{\sqrt[3]{x+9}}.$$

List the intervals where f is increasing and list as well as classify all critical values.

$f'(x)$ is undefined for $x=0,-9$. And $f'(x)=0$ when $x=-2, \pm 1$

		-9		-2		-1		0		1	
$f'(x)$	$-$		$+$		$+$		$-$		$-$		$+$
$f(x)$	↘		↗		↗		↘		↘		↗

rel. max at $x=-1$
rel. min at $x=-9, 1$
at $x=0,-2$ no extrema

$f(x)$ is increasing on $(-9,-2)$, $(-2,-1)$, $(1,\infty)$

[8] 4. Find the global maximum and the global minimum of $f(x) = (x^2-2x)e^x$ on the interval $[0,3]$. For **5 bonus marks** find the global maximum and minimum of f on $(-\infty, 0]$ or explain why they do not exist.

$f'(x) = (2x-2)e^x + (x^2-2x)e^x = (x^2-2)e^x$. $f'(x)$ is defined on $[0,3]$ and $f'(x)=0$ when $x=\pm\sqrt{2}$. Critical points on $[0,3]$ are $x=\sqrt{2}$.

Now $f(\sqrt{2}) = (2-2\sqrt{2})e^{\sqrt{2}} = 2(1-\sqrt{2})e^{\sqrt{2}} < 0$ (since $(1-\sqrt{2})<0$)

$f(0)=0$

$f(3) = 3e^3 > 0$

So we have abs max of $3e^2$ when $x=3$ and abs min of $(2-2\sqrt{2})e^{\sqrt{2}}$ when $x=\sqrt{2}$

Bonus

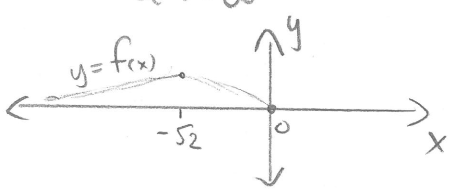
Consider $f'(x) = (x^2-2)e^x$ on $(-\infty, 0]$. Note that $f(0)=0$.
From above work we know $x=-\sqrt{2}$ is critical point.

		$-\sqrt{2}$		0
$f'(x)$	$+$		$-$	
$f(x)$	↗		↘	

So we have a local max at $x=-\sqrt{2}$ and

$f(-\sqrt{2}) = \frac{2(1+\sqrt{2})}{e^{\sqrt{2}}} > 0$

Now $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (x^2-2x)e^x = \lim_{x\to\infty} \frac{x^2+2x}{e^x} = 0 \Rightarrow$ horizontal asymptote $y=0$



Therefore we have global min of 0 at $x=0$ and global max of $\frac{2(1+\sqrt{2})}{e^{\sqrt{2}}}$ at $x=-\sqrt{2}$

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