Math 1210: Worksheet #6 **Fall 2017**

Name: Solution	A#:	Section: H

[6] 1. Consider a right-angled triangle with catheti (adjacent sides) x and y and hypothenuse (opposite side) z. When $x = 5$ and $y = 12$ we have that x is growing at the rate of $7cm/s$ and the angle θ between x and z is growing at the rate of $\frac{1}{4}rad/s$, (the right angle $\frac{\pi}{2}$ between x and y is fixed throughout the process). What is the rate of change of z at that point? For **2 bonus marks** determine the rate of change of y at that point.

$\frac{dx}{dt} = 7\ cm/s$

$\frac{d\theta}{dt} = 1/4\ rad/s$ $\Rightarrow$ $x = 5,\ y = 12 \rightarrow z^2 = x^2 + y^2$

$z^2 = 25 + 144$

$\boxed{z = 13}$

$\cos\theta = x/z$ relates x and z. Take the derivitive w.r.t. t

$$\frac{d}{dt}(\cos\theta) = \frac{d}{dt}\left(\frac{x}{z}\right) \Rightarrow \boxed{-\sin\theta \frac{d\theta}{dt} = \frac{\frac{dx}{dt} z - \frac{dz}{dt} x}{z^2}}$$

we want to find $\frac{dz}{dt} \Rightarrow \frac{-12}{13} \cdot \frac{1}{4} = \frac{7 \cdot 13 - \frac{dz}{dt} \cdot 5}{13^2}$

$$-\frac{3}{13} = \frac{91 - 5\frac{dz}{dt}}{169} \Rightarrow -39 = 91 - 5\frac{dz}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{130}{5} = 26\ cm/s$$

Bonus

$$\frac{d}{dt}(z^2 = x^2 + y^2) \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\Rightarrow 2 \cdot 13 \cdot 26 = 2 \cdot 5 \cdot 7 + 2 \cdot 12 \frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{\overbrace{13 \cdot 26}^{338} - 35}{12} = \boxed{\frac{303}{12}}$$

[3] 2. Let x and y be functions of t related by $x^2 + y^2 = xy + 7$. When $x = 3$ and $y = 2$ we have that $\frac{dx}{dt} = -2$. Find the value of $\frac{dy}{dt}$ at that point.

$$\frac{d}{dt}(x^2 + y^2 = xy + 7)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = \frac{dx}{dt} y + x\frac{dy}{dt}$$

$$\xrightarrow{x=3, y=2} 2(3)(-2) + 2(2)\left(\frac{dy}{dt}\right) = (-2)(2) + (3)\left(\frac{dy}{dt}\right)$$

$\frac{dx}{dt} = -2$

$$-12 + 4\frac{dy}{dt} = -4 + 3\frac{dy}{dt}$$

$$\boxed{\frac{dy}{dt} = 8}$$

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[4] 3. Let f be a function whose **derivative** is

$$f'(x) = \frac{(x+2)^4 \ln(x^2) e^x}{\sqrt[3]{x+9}}.$$

List the intervals where f is increasing and list as well as classify all critical values.

- $f'(x)$ is undefined for $x = 0, -9$. ($\ln(x^2)$ is undefined for $x=0$)
- $f'(x) = 0$ when $x = 2, \pm 1$ ($\ln(x^2) = 0$ when $x^2 = 1 \to x = \pm 1$)

		-9		-2		-1		0		1	
$f'(x)$	$-$		$+$		$+$		$-$		$-$		$+$
$f(x)$	↘		↗		↗		↘		↘		↗

$\boxed{f(x) \text{ is increasing on } (-9,-2), (-2,-1), (1, \infty)}$

$\boxed{\text{local max at } x=-1 \quad \text{local min at } x = -9, 1}$

$\boxed{\text{No extrema at } x=0 \;\&\; x=-2}$

[7] 4. Find the global maximum and the global minimum of $f(x) = (x^2 - 2x)e^x$ on the interval $[0, 3]$. For **3 bonus marks** find the global maximum and minimum of f on $(-\infty, 0]$ or explain why they do not exist.

$f'(x) = (2x-2)e^x + (x^2-2x)e^x$

$= (x^2-2)e^x \implies f'(x) = 0 \quad \boxed{x = \pm\sqrt{2}}$

$f'(x)$ is defined on $[0,1]$ and $f'(x) = 0$ when $x = \pm\sqrt{2}$

The critical points on $[0,3]$ are $x = \sqrt{2}$

$f(\sqrt{2}) = (2-2\sqrt{2})e^{\sqrt{2}} = 2(1-\sqrt{2})e^{\sqrt{2}} < 0$ (since $(1-\sqrt{2}) < 0$)

$f(0) = 0$

$f(3) = 3e^3 > 0$

$\implies$ $\boxed{\text{So } f \text{ has abs. max of } 3e^2 \text{ when } x=3 \text{ and abs. min of } (2-2\sqrt{2})e^{\sqrt{2}} \text{ when } x = \sqrt{2}}$

Bonus

Consider $f'(x) = (x^2-2)e^x$ on $(-\infty, 0]$. Note that $f(0) = 0$

$f'(x) = 0 \to x = \pm\sqrt{2}$ So $x = -\sqrt{2}$ is critical point.

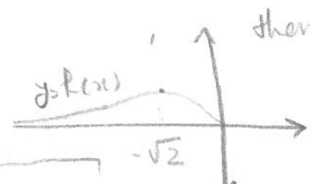
		$-\sqrt{2}$		0
$f'(x)$	$+$		$-$	
$f(x)$	↗		↘	

So we have a local max at $x = -\sqrt{2}$

and $f(-\sqrt{2}) = \frac{2+2\sqrt{2}}{e^{\sqrt{2}}} > 0$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (x^2-2x)e^x = \lim_{x \to \infty} \frac{x^2+2x}{e^x} = 0 \implies$ as exponential grows much faster than polynemial.

$\implies$ $\boxed{\text{horizontal asymptote } y = 0}$



$\boxed{\text{Therefore, we have global min of 0 at } x=0 \text{ and global max of } \frac{2+2\sqrt{2}}{e^{\sqrt{2}}} \text{ at } x = -\sqrt{2}}$

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