Math 1210: Worksheet #7 **Fall 2017**

Name: Solutions (Micah)	A#:	Section:

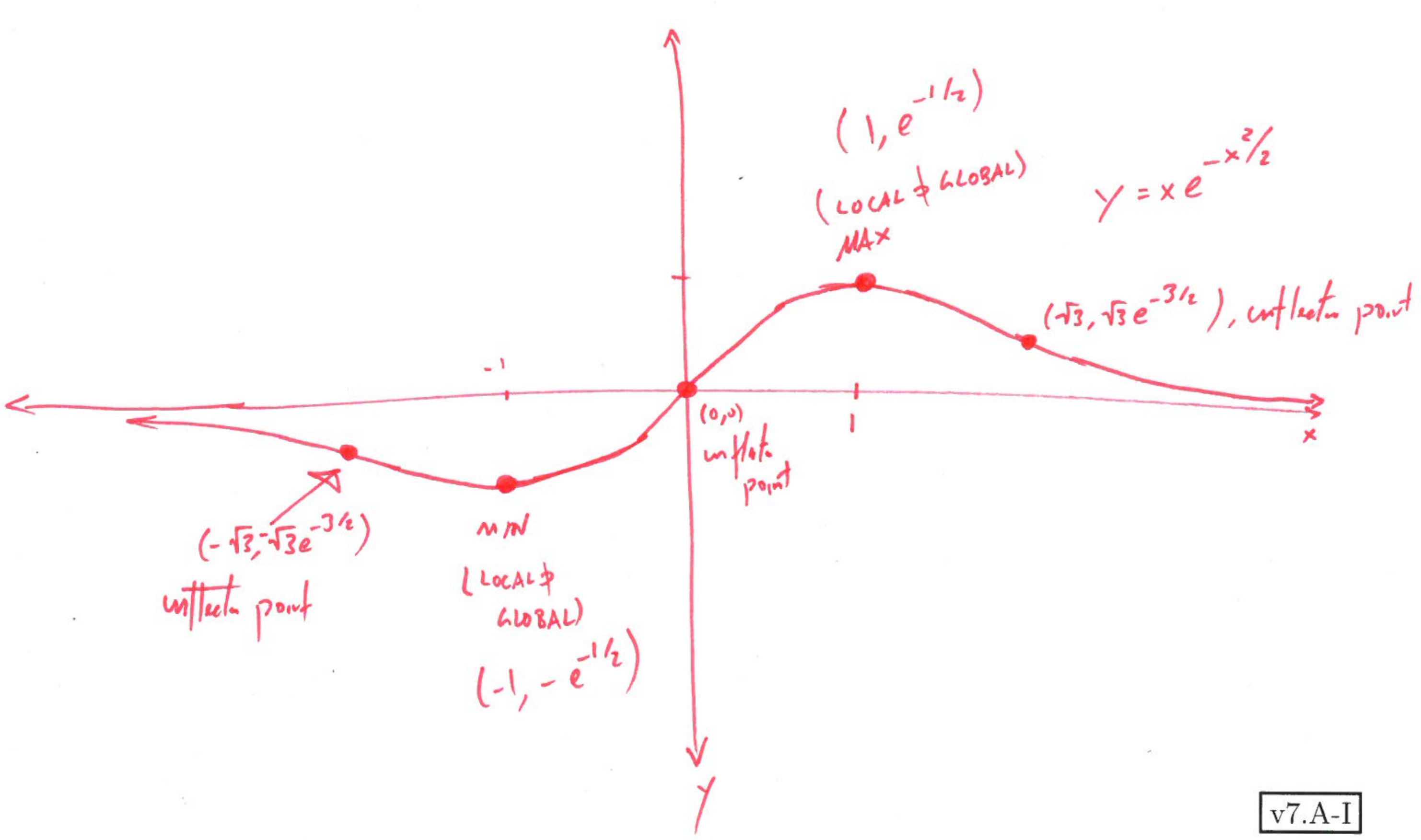
[8] 1. Sketch the graph of $y = xe^{-x^2/2}$. You can use $y' = (1-x^2)e^{-x^2/2}$ and $y'' = x(x^2-3)e^{-x^2/2}$. Make sure that your graph clearly shows any x and y intercepts, horizontal asymptotes, vertical asymptotes, local extrema, intervals where the function is increasing and where it is decreasing, concavity, and inflection points.

$\lim_{x\to+\infty} xe^{-x^2/2} = 0$, $\lim_{x\to-\infty} xe^{-x^2/2} = 0$, there are horizontal asymptotes.

when $x=0$, $y=0$, this is the only time $y=0$. The function is odd.

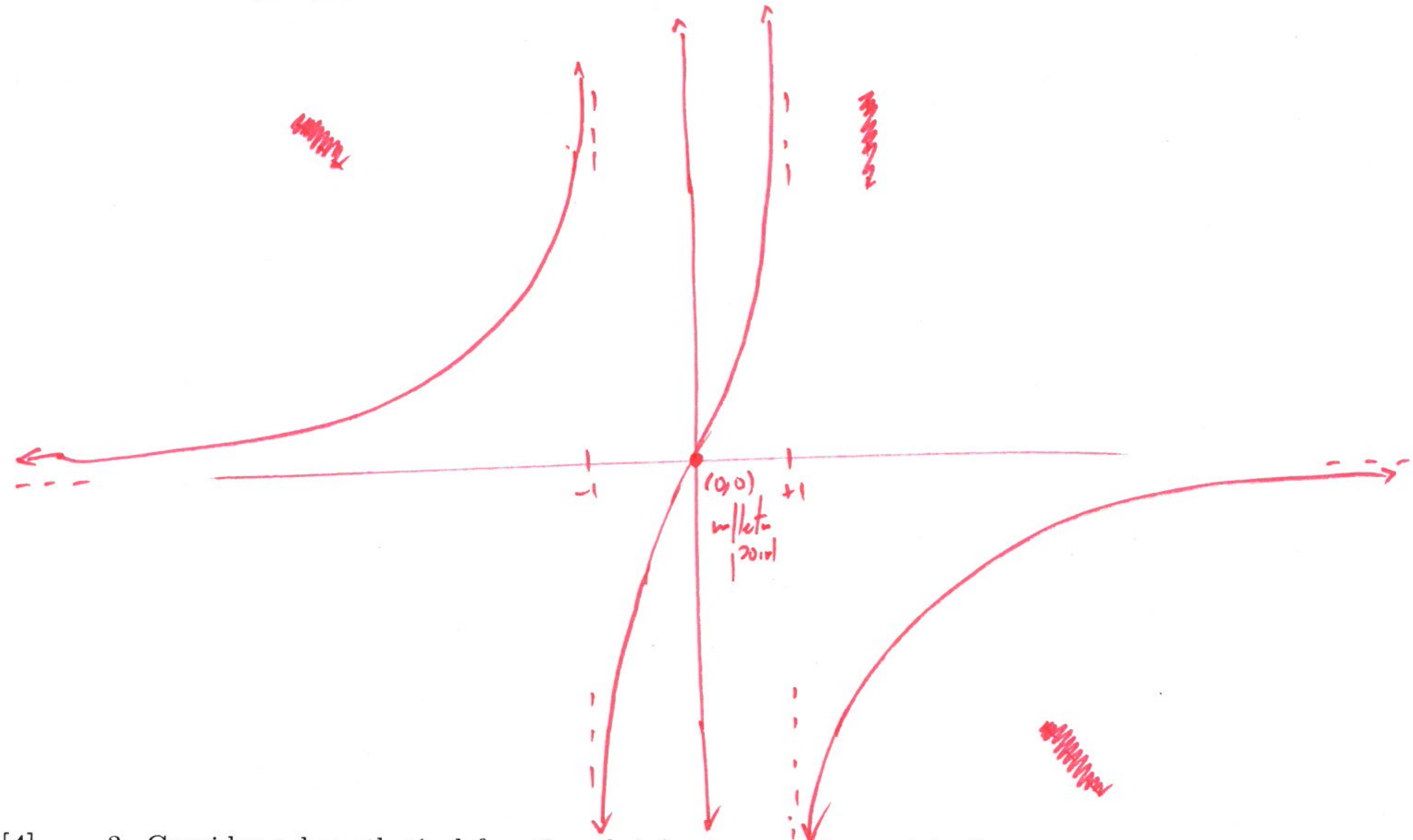
y' is everywhere defined, hence critical points occur when $y'=0$, that is, $x=\pm 1$!

y' is positive when $x \in (-1,+1)$ and negative on $(-\infty,-1)$ and $(1,\infty)$, hence y is increasing on $(-1,+1)$ and decreasing on $(-\infty,-1)$ and $(1,\infty)$. Candidates for inflection points are when $y''=0$, that is, when $x = 0, \pm\sqrt{3}$. Since $y''>0$ when $x\in(-\sqrt{3},0)$ and $x \in (\sqrt{3},\infty)$, hence y is concave up there; similarly y is concave down on $(-\infty,-\sqrt{3})$ and $(0,\sqrt{3})$ since $y''<0$ on those intervals. Hence we see that $0,\pm\sqrt{3}$ are inflection points and that $+1$ is a max and that -1 is a min.



[8] 2. Sketch the graph of $y = \frac{x}{1-x^2}$. You can use $y' = \frac{1+x^2}{(1-x^2)^2}$ and $y'' = \frac{2x(x^2+3)}{(1-x^2)^3}$. Make sure that your graph clearly shows any x and y intercepts, horizontal asymptotes, vertical asymptotes, local extrema, intervals where the function is increasing and where it is decreasing, concavity, and inflection points.

As $x \to +\infty$, $y \to 0$, and as $x \to -\infty$, $y \to 0$. When $x = 0$, $y = 0$, this is the only place where $y = 0$. y is undefined at $x = \pm 1$ and has vertical asymptotes there. Since $y' > 0$ for all x except ± 1 where it is undefined, it is increasing on $(-\infty, -1)$ and $(-1, +1)$ and $(1, \infty)$ and has critical points at $x = \pm 1$. $y'' = 0$ has roots $x = 0$ so this is the only candidate for an inflection point, since $y'' > 0$ when $x \in (-\infty, -1)$ and $(0, 1)$, y is concave up on these intervals, similarly it is concave down on $(-1, 0)$ and $(1, +\infty)$ since $y'' < 0$ on those intervals. We see that $x = 0$ is an inflection point.



[4] 3. Consider a hypothetical function f defined everywhere with the property that f has a discontinuity at 3 and critical points at $-5, -3, 1, 7, 9$. Find the global maximum and global minimum, or state that they do not exists, of f on $I = [0, \infty)$ for the following cases.

(a)

$\lim_{x\to-\infty} f(x)$	$f(-5)$	$f(-3)$	$f(0)$	$f(1)$	$\lim_{x\to3^-} f(x)$	$f(3)$	$\lim_{x\to3^+} f(x)$	$f(7)$	$f(9)$	$\lim_{x\to\infty} f(x)$
∞	-42	15	7	0	-7	-5	9	10	4	3

Maximum $x = 7$, $f(7) = 10$ **Minimum** DOES NOT EXIST

(b)

$\lim_{x\to-\infty} f(x)$	$f(-5)$	$f(-3)$	$f(0)$	$f(1)$	$\lim_{x\to3^-} f(x)$	$f(3)$	$\lim_{x\to3^+} f(x)$	$f(7)$	$f(9)$	$\lim_{x\to\infty} f(x)$
$-\infty$	-42	5	5	7	5	9	9	10	14	∞

Maximum DOES NOT EXIST **Minimum** $x = 0$, $f(0) = 5$