Math 1210: Worksheet #8 **Fall 2017**

Name: SOLUTION	A#:	Section:

[5] 1. Find the linearisation $L(x)$ of $f(x) = \tan^{-1}(x)$ centred at 1.

$$f'(x) = \frac{1}{1+x^2}, \quad f'(1) = \frac{1}{2}, \quad f(1) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\boxed{L(x) = \frac{\pi}{4} + \frac{1}{2}(x-1)}$$

[5] 2. Estimate $\sqrt{23}$. Is your estimate larger or smaller then $\sqrt{23}$? Justify your answer.

$$f(x) = \sqrt{x}, \quad a = 25, \quad f'(x) = \frac{1}{2\sqrt{x}}, \quad f(25) = 5, \quad f'(25) = \frac{1}{10}$$

$$\boxed{L(x) = 5 + \frac{1}{10}(x-25)}$$

$$\sqrt{23} \approx L(23) = 5 + \frac{1}{10}(23-25) = 5 - \frac{2}{10} = \boxed{\frac{24}{5}}$$

$$f''(x) = \frac{d}{dx}\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = -\frac{1}{4}x^{-3/2}$$. This is negative on $[23, 25]$,

so $\boxed{\frac{24}{5} > \sqrt{23}}$

[5] 3. Find the point(s) on the curve $y = \sqrt{x+1}$, $x \geq -1$, that is closest to the point $(0, 0)$. (Hint: minimize the square of the distance)

$f(x) = \text{Distance}^2 = x^2 + y^2 = x^2 + x + 1$.

$f'(x) = 2x + 1$. Critical point $= -\frac{1}{2}$ $\quad f' \; - \;\big|\; +$, $-\frac{1}{2}$

$f(-1) = 1$, $f(-\frac{1}{2}) = \frac{3}{4}$, $\lim_{x\to\infty} f(x) = \infty$

(or f has a local min at $-\frac{1}{2}$ & $-\frac{1}{2}$ is the only crit. point).

So $\boxed{(-\frac{1}{2}, \sqrt{\frac{1}{2}})}$ is the closest point.

[5] 4. Find the point(s) (x, y) on the curve $y = -x^3 + 8x^2 - 10x$ with largest value of $P = xy$.

$P = xy = x(-x^3 + 8x^2 - 10x) = -x^4 + 8x^3 - 10x^2$

$P' = -4x^3 + 24x^2 - 20x = -4x(x^2 - 6x + 5) = -4x(x-5)(x-1)$.

Critical points: 0, 1, 5

$\lim_{x\to-\infty} P(x) = -\infty$, $P(0) = 0$, $P(1) = -3$, $P(5) = 125$, $\lim_{x\to\infty} P(x) = -\infty$.

Maximum is at $x = 5$, the point is $\boxed{(5, 25)}$.