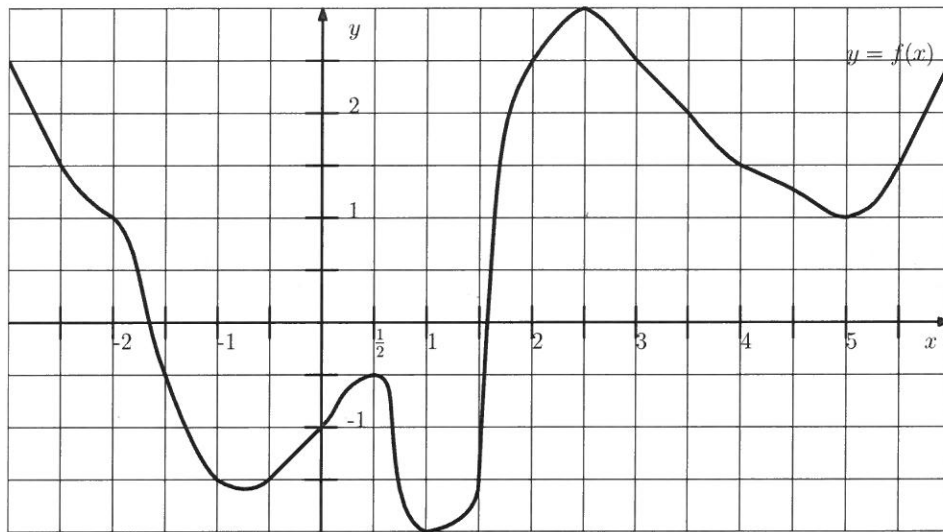


Name: SOLUTION

A#:

Section:

- [7] 1. Let f be a function whose graph of $y = f(x)$ is given below. Let L_N and R_N denote the left-endpoint, respectively right end-point, Riemann sums with N subintervals of equal size. Fill in the following.



- (a) Estimate the area under the curve over $[2, 5]$ by L_2 : $\frac{3}{2} \left(\frac{5}{2} + 2 \right) = \frac{27}{4}$
- (b) Estimate the area under the curve over $[2, 4]$ by R_4 : $\frac{2}{4} \left(3 + \frac{5}{2} + 2 + \frac{3}{2} \right) = \frac{18}{4} = \frac{9}{2}$
- (c) Estimate $\int_{-1}^2 f(x) dx$ by L_2 : $\frac{3}{2} \left(-\frac{3}{2} - \frac{1}{2} \right) = -3$
- (d) Estimate $\int_0^3 \left(f\left(\frac{x}{2}\right) + 1 \right)^2 dx$ by R_3 : $1 \cdot \left(\left(f\left(\frac{1}{2}\right) + 1 \right)^2 + \left(f\left(\frac{2}{2}\right) + 1 \right)^2 + \left(f\left(\frac{3}{2}\right) + 1 \right)^2 \right) = \frac{1}{4} + 1 + \frac{1}{4} = \frac{3}{2}$
- (e) If $F(x) = \int_0^x f(t) dt$, then $F'(3) = f(3) = \frac{5}{2}$
- (f) If $G(x) = \int_{-1}^{x^2} t^2 f(t+1) dt$, then $G'(x) = 2x \left((x^2)^2 f(x^2+1) \right) = 2x^5 f(x^2+1)$
and $G'(2) = 2(2)^5 f(5) = 64$

[4] 2. Solve the initial value problem $\frac{dy}{dx} = 3e^{2x}$, $y(0) = 4$.

$$y = \int 3e^{2x} dx = \frac{3}{2}e^{2x} + C, \quad 4 = y(0) = \frac{3}{2}e^0 + C \Rightarrow C = 4 - \frac{3}{2} = \frac{5}{2}$$

$$\boxed{y = \frac{3}{2}e^{2x} + \frac{5}{2}}$$

3. Compute the integral.

[5] (a) $\int \left(\sin(2x) + \frac{2}{1+x^2} + \sec^2(3x) + e^3 + x^{-3} \right) dx$

$$= -\frac{1}{2} \cos(2x) + 2 \tan^{-1}(x) + \frac{1}{3} \tan(3x) + e^3 x + \frac{x^{-2}}{-2} + C$$

[4] (b) $\int_1^2 \frac{1+t^2}{t} dt = \int_1^2 \left(\frac{1}{t} + t \right) dt = \left(\ln|t| + \frac{t^2}{2} \right) \Big|_1^2 = \left(\ln 2 + \frac{2^2}{2} \right) - \left(\ln 1 + \frac{1^2}{2} \right)$

$$= \boxed{(\ln 2) + \frac{3}{2}}$$