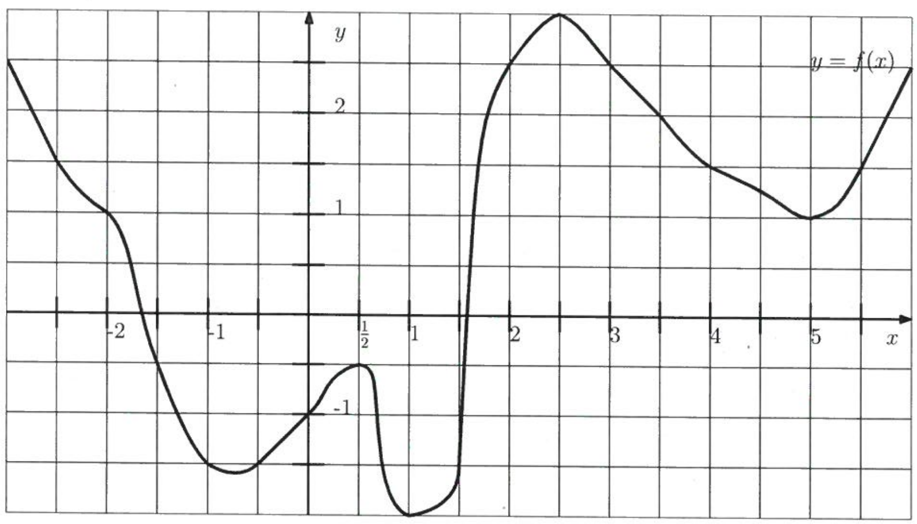
Math 1210: Quiz #9 — Fall 2017

Name: Mini (solutions)	A#:	Section:

[7] 1. Let f be a function whose graph of $y = f(x)$ is given below. Let L_N and R_N denote the left-endpoint, respectively right end-point, Riemann sums with N subintervals of equal size. Fill in the following.



(a) Estimate the area under the curve over $[2, 5]$ by L_2: $(1.5)\left(f(2) + f(3.5)\right) = \frac{3}{2}\left(\frac{5}{2} + 2\right) = \frac{27}{4}$

(b) Estimate the area under the curve over $[3, 5]$ by R_4: $\left(\frac{1}{2}\right)\left(f(3.5) + f(4) + f(4.5) + f(5)\right) = \frac{1}{2}\left(2 + \frac{3}{2} + \frac{5}{4} + 1\right)$ $= 23/8$

(c) Estimate $\int_{-1}^{2} f(x)dx$ by L_2: $\frac{3}{2}\left(f(-1) + f(0.5)\right) = \frac{3}{2}\left(-1 - \frac{1}{2}\right) = -9/4$

(d) Estimate $\int_0^3 \left(f\left(\frac{x}{2}\right) + 1\right)^2 dx$ by R_3: $1\left[\left(f\left(\frac{1}{2}\right)+1\right)^2 + \left(f\left(\frac{3}{2}\right)+1\right)^2 + \left(f\left(\frac{5}{2}\right)+1\right)^2\right]$ $= 0^2 + (-1)^2 + (-1/2)^2 = 1 + 1/4$ $= 5/4$

(e) If $F(x) = \int_0^x f(t)dt$, then $F'(3) =$ $f(3) = 2.5$

(f) If $G(x) = \int_{-1}^{x^2} t^2 f(t+1)dt$, then $G'(x) =$ $2x^5 f(x^2+1)$

and $G'(2) =$ $64f(5) = 64$

[4] 2. Solve the initial value problem $\frac{dy}{dx} = 3e^{2x}$, $y(0) = 2$.

$$y = \int 3e^{2x}\,dx = \frac{3e^{2x}}{2} + c$$

$$y(0) = \frac{3}{2} + c = 2 \quad \text{so} \quad c = 1/2$$

Hence $y = \dfrac{3e^{2x}}{2} + \dfrac{1}{2}$

3. Compute the integral.

[5] (a) $\displaystyle\int \left(\sin(2x) + \frac{2}{1+x^2} + \sec^2(3x) + e^4 + x^{-2}\right) dx$

$$= \frac{-\cos(2x)}{2} + 2\tan^{-1}x + \frac{\tan 3x}{3} + e^4 x - \frac{x^{-1}}{2}$$

[4] (b) $\displaystyle\int_1^4 \frac{1+t^2}{t} dt = \int_1^4 \left(\frac{1}{t} + t\right) dt = \left[\ln t + \frac{t^2}{2}\right]_1^4$

$$= \ln 4 + 8 - \ln 1 - 1/2 = \ln 4 + 7/2$$