Math 1210: Worksheet #9 **Fall 2017**

Name: SOLUTION	A#:	Section:

[2] 1. Suppose that $F(x) = x^x$ is an antiderivative of $f(x)$. Find $f(x)$.

$$f(x) = F'(x) = \frac{d}{dx} e^{x \ln x} = e^{x \ln x}\left(\ln x + x \cdot \frac{1}{x}\right) = x^x(\ln x + 1).$$

[6] 2. Solve the initial value problem $y'' = \frac{x^2+\sqrt{x}}{x}$, $y(1) = 1$, $y'(1) = 0$.

$$y' = \int y \, dx = \int (x + x^{-\frac{1}{2}}) dx = \frac{1}{2}x^2 + 2x^{\frac{1}{2}} + C.$$

$$0 = y'(1) = \frac{1}{2} + 2 + C \quad \therefore \quad C = -\frac{5}{2}.$$

$$y' = \frac{1}{2}x^2 + 2x^{\frac{1}{2}} - \frac{5}{2}.$$

$$y = \int y' dx = \int \left(\frac{1}{2}x^2 + 2x^{\frac{1}{2}} - \frac{5}{2}\right) dx = \frac{1}{6}x^3 + \frac{4}{3}x^{\frac{3}{2}} - \frac{5}{2}x + D.$$

$$1 = y(1) = \frac{1}{6} + \frac{4}{3} - \frac{5}{2} + D = -1 + D \quad \therefore \quad D = 2.$$

$$\boxed{y = \frac{1}{6}x^3 + \frac{4}{3}x^{3/2} - \frac{5}{2}x + 2}$$

[3] 3. If $F(x) = \int_{-\ln x}^{\sin x} e^{t^2} dt$, then compute $F'(x)$.

$$F'(x) = -\left(-\frac{1}{x}\right)e^{(-\ln x)^2} + \cos x \, e^{(\sin x)^2} = \frac{1}{x} e^{(\ln x)^2} + \cos x \, e^{\sin^2 x}$$

[9] 4. Compute the integral.

(a) $\int \frac{1}{\sqrt{2x - x^2}} dx$ (Hint: complete the square)

$$= \int \frac{1}{\sqrt{1-(x-1)^2}}\, dx = \sin^{-1}(x-1) + C$$

(b) $\int_0^1 \frac{e^{2x} - e^{-2x}}{e^{x+1}} dx$ $= \int_0^1 (e^{x-1} - e^{-3x-1})\, dx = \left(e^{x-1} + \frac{1}{3} e^{-3x-1}\right)\Big|_0^1$

$$= \left(e^0 + \frac{1}{3} e^{-4}\right) - \left(e^{-1} + \frac{1}{3} e^{-1}\right) = \boxed{1 - \frac{4}{3} e^{-1} + \frac{1}{3} e^{-4}}$$

(c) $\int_1^{\sqrt{3}} \frac{1}{3 + x^2} dx$ $= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\Big|_1^{\sqrt{3}} = \frac{1}{\sqrt{3}} \tan^{-1}(1) - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \boxed{\frac{\pi}{12\sqrt{3}}}$$

by inspection, or $\int \frac{1}{3+x^2} dx = \frac{1}{3} \int \frac{1}{1+\left(\frac{x}{\sqrt{3}}\right)^2} dx = \frac{1}{3} \cdot \left(\frac{1}{\frac{1}{\sqrt{3}}}\right) \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C.$$

v9.AB