Math 1210: Worksheet #9 **Fall 2017**

Name:	A#:	Section:

[2] 1. Suppose that $F(x) = x^x(1 + \ln x)$ is an antiderivative of $f(x)$. Find $f(x)$.

$$f(x) = F'(x) = \frac{d}{dx}\left(e^{x\ln x}(1+\ln x)\right) = \frac{e^{x\ln x}}{x} + (1+\ln x)\left(e^{x\ln x}(\ln x+1)\right)$$

$$= x^x\left(\frac{1}{x} + (1+\ln x)^2\right)$$

[6] 2. Solve the initial value problem $y'' = \frac{x^2+\sqrt{x}}{x}$, $y(1) = 1$, $y'(1) = 0$.

$$y' = \int y''\,dx = \int (x + x^{-1/2})\,dx = \frac{x^2}{2} + 2x^{1/2} + C$$

$$y'(1) = \frac{1}{2} + 2 + C = 0 \quad \text{so} \quad C = -5/2$$

$$y = \int y'\,dx = \int\left(\frac{x^2}{2} + 2x^{1/2} - 5/2\right)dx$$

$$= \frac{x^3}{6} + \frac{4}{3}x^{3/2} - \frac{5x}{2} + D$$

$$y(1) = \frac{1}{6} + \frac{4}{3} - \frac{5}{2} + D = 1 \quad \text{so } D = 2$$

$$y = \frac{x^3}{6} + \frac{4}{3}x^{3/2} - \frac{5x}{2} + 2$$

[3] 3. If $F(x) = \int_{-\ln x}^{\sin x} e^{t^2}\,dt$, then compute $F'(x)$.

Let $G(x)$ be such that $G'(x) = e^{x^2}$, then

$$F(x) = G(\sin x) - G(-\ln x)$$

Hence ~~$F'(x) = G'(\sin x) - G'(-\ln x)$~~

Hence $F'(x) = \cos x \cdot G'(\sin x) + \frac{1}{x} G'(-\ln x)$

$$= \cos x \cdot e^{\sin^2 x} + \frac{e^{(\ln x)^2}}{x}$$

[9] 4. Compute the integral.

(a) $\int \frac{1}{\sqrt{2x - x^2}} dx$ (Hint: complete the square)

$$= \int \frac{1}{\sqrt{1-(x-1)^2}} dx = \sin^{-1}(x-1) + C$$

(b) $\int_0^1 \frac{e^{2x} - e^{-2x}}{e^{x+1}} dx$

$$= \int_0^1 \left(e^{x-1} - e^{-3x-1}\right) dx$$

$$= \left(e^{x-1} + \frac{e^{-3x-1}}{3}\right)\Bigg|_0^1 = \left(e^0 + \frac{e^{-4}}{3}\right) - \left(e^{-1} + \frac{e^{-1}}{3}\right)$$

$$= 1 + \frac{1}{3e^4} - \frac{4}{3e}$$

(c) $\int_1^{\sqrt{3}} \frac{1}{3 + x^2} dx$

$$= \frac{1}{3}\int_1^{\sqrt{3}} \frac{1}{1+(x/\sqrt{3})^2} dx = \frac{\sqrt{3}}{3} \tan^{-1}(x/\sqrt{3})\Big|_1^{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}\left(\tan^{-1} 1 - \tan^{-1}(1/\sqrt{3})\right) = \frac{\sqrt{3}}{3}\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{3}\left(\frac{\pi}{12}\right) = \frac{\pi\sqrt{3}}{36}$$

v9.CD