Math 1210: Worksheet #10 Fall 2017

Name: Solution	A#:	Section: H

[3] 1. $\int_0^{\frac{\pi}{4}} \sin(x)\sec^6(x)dx$

⊛ let $u = \cos(x)$

$du = -\sin(x)dx$

when $x = \pi/4$, $u = \cos(\pi/4) = 1/\sqrt{2}$

when $x = 0$, $u = \cos(0) = 1$

$$\overset{⊛}{=} \int_1^{1/\sqrt{2}} \frac{1}{u^6}\, du$$

$$= \int_1^{1/\sqrt{2}} -u^{-6}\, du$$

$$= \frac{1}{5} u^{-5} \Big|_1^{1/\sqrt{2}} = \frac{1}{5}(\sqrt{2})^5 - \frac{1}{5}$$

$$= \boxed{\frac{1}{5}(4\sqrt{2}-1)}$$

Note: sometimes there is more than one way to solve the question. For the question abov $u = \sec(x)$ works, as well.

[3] 2. $\int_0^{\ln(2)} \frac{e^{2x}}{e^{2x}+1}dx$

⊛ let $u = e^{2x}+1$

$du = 2e^{2x}dx \Rightarrow \frac{1}{2}du = e^{2x}dx$

when $x = \ln(2)$, $u = e^{2\ln(2)}+1$

$= e^{\ln(2^2)}+1$

$= 5$

when $x = 0$, $u = e^0 + 1 = 2$

$$\overset{⊛}{=} \frac{1}{2}\int_2^5 \frac{1}{u}\, du$$

$$= \frac{1}{2}\ln(u)\Big|_2^5$$

$$= \frac{1}{2}(\ln(5)-\ln(2)) = \boxed{\frac{1}{2}\ln(5/2)}$$

[3] 3. $\int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{e^{2x}+1}dx$

⊛ let $u = e^x$

$du = e^x dx$

when $x = \frac{1}{2}\ln(3) = \ln(3^{1/2}) = \ln(\sqrt{3})$,

$u = e^{\ln(\sqrt{3})} = \sqrt{3}$

when $x = 0$, $u = e^0 = 1$

$$\overset{⊛}{=} \int_1^{\sqrt{3}} \frac{1}{u^2+1}\, du$$

$$= \tan^{-1}(u)\Big|_1^{\sqrt{3}}$$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$$

[3] 4. $\int_0^1 \frac{x^2-1}{\sqrt{1+x}}dx = \int_0^1 \frac{(x+1)(x-1)}{\sqrt{1+x}}dx$

$\circledast$ Let $u = \sqrt{x+1} \Rightarrow u^2 = x+1$

$\Rightarrow u^2 - 2 = x - 1$

$du = \frac{1}{2\sqrt{x+1}} \Rightarrow 2du = \frac{1}{\sqrt{x+1}}$

$x = 1 \longrightarrow u = \sqrt{1+1} = \sqrt{2}$

$x = 0 \longrightarrow u = \sqrt{0+1} = 1$

$\overset{\circledast}{=} \int_1^{\sqrt{2}} (u)^2(u^2-2)\cdot 2du$

$= 2\int_1^{\sqrt{2}} (u^4 - 2u^2)\,du$

$= 2\left[\left(\frac{1}{5}u^5 - \frac{2}{3}u^3\right)\Big|_1^{\sqrt{2}}\right]$

$= 2\left(\frac{1}{5}\, 2^{5/2} - \frac{2}{3}\, 2^{3/2} - \frac{1}{5}(1) + \frac{2}{3}(1)\right) = \frac{3(2^{7/2}) - 5(2^{7/2}) - 6 + 20}{15} = \frac{14 - 2^{9/2}}{15}$

$\boxed{\frac{14 - 8\sqrt{2}}{15}}$

[4] 5. $\int \frac{x+2}{x^2+4}dx = \overset{①}{\int \frac{x}{x^2+4}dx} + \overset{②}{\int \frac{2}{x^2+4}dx} = \boxed{\frac{1}{2}\ln(x^2+4) + \tan^{-1}(x/2) + C}$

① $\int \frac{x}{x^2+4}dx$ $\circledast$ let $u = x^2+4$

$du = 2x$

$\frac{1}{2}du = x$

$\overset{\circledast}{=} \frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}\ln u + C$

$= \frac{1}{2}\ln(x^2+4) + C$

② $\int \frac{2}{x^2+4}dx$

$\circledast\circledast$ let $u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$\overset{\circledast\circledast}{=} \int \frac{\frac{1}{2}}{(\frac{x}{2})^2+1}dx$

$= \int \frac{1}{u^2+1}du = \tan^{-1}(u) + C$

$= \tan^{-1}(x/2) + C$

[4] 6. $\int \sqrt{1+\sqrt{x}}dx$

$\circledast$ let $u = 1 + \sqrt{x} \longrightarrow \sqrt{x} = u - 1$

$2\sqrt{x} = 2u - 2$ $\circledast$

$du = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,du = dx$

$\circledast$ $(2u-2)du = dx$

$\overset{\circledast}{=} \int \sqrt{u}\,(2u-2)du$

$= 2\int (u^{3/2} - u^{1/2})du$

$= \frac{4}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$

$\boxed{= \frac{4}{5}(1+\sqrt{x})^{5/2} - \frac{4}{3}(1+\sqrt{x})^{3/2} + C}$