Math 1210: Worksheet #10 — Fall 2017

Name: solution	A#:	Section:

[3] 1. $\int_0^{\frac{\pi}{4}} \sin(x)\sec^6(x)dx$

$= \int_0^{\pi/4} \tan(x)\sec^5(x)\,dx$ $\quad \begin{cases} \text{let } u = \sec x \\ du = \sec x \tan x\, dx \end{cases}$ $\quad \begin{cases} x = 0 \to u = \sec(0) = 1 \\ x = \frac{\pi}{4} \to u = \sec(\pi/4) = \sqrt{2} \end{cases}$

$= \int_0^{\pi/4} \sec^4(x)\sec(x)\tan(x)\,dx$

$= \int_1^{\sqrt{2}} u^4\,du$

$= \left.\frac{u^5}{5}\right]_1^{\sqrt{2}}$

$= \frac{1}{5}\left((\sqrt{2})^5 - 1\right) = \frac{1}{5}\left(4\sqrt{2} - 1\right)$

[3] 2. $\int_0^{\ln(2)} \frac{e^{2x}}{e^{2x}+1}dx = \int_0^{\ln(2)} \frac{e^{2x}}{e^{2x}+1}\,dx$ $\quad \begin{cases} \text{let } u = e^{2x}+1 \\ du = 2e^{2x}dx \rightsquigarrow \frac{du}{2} = e^{2x}dx \end{cases}$ $\quad \begin{cases} x = 0 \to u = e^0 + 1 = 2 \\ x = \ln 2 \to u = e^{2\ln 2} + 1 = 5 \end{cases}$

$= \int_2^5 \frac{1}{u}\,\frac{du}{2}$

$= \frac{1}{2}\int_2^5 \frac{1}{u}\,du$

$= \frac{1}{2}\left[\ln|u|\right]_2^5$

$= \frac{1}{2}\left[\ln 5 - \ln 2\right] = \boxed{\frac{1}{2}\ln(5/2)}$

[3] 3. $\int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{e^{2x}+1}dx = \int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{(e^x)^2+1}\,dx$ $\quad \begin{cases} \text{let } u = e^x \\ du = e^x dx \end{cases}$ $\quad \begin{cases} x = 0 \to u = e^0 = 1 \\ x = \frac{1}{2}\ln(3) \to u = e^{\frac{1}{2}\ln(3)} = \sqrt{3} \end{cases}$

$= \int_1^{\sqrt{3}} \frac{1}{u^2+1}\,du$

$= \left[\tan^{-1}(u)\right]_1^{\sqrt{3}}$

$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$

$= \frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$

or $u = x+1$ also works!

[3] 4. $\int_0^1 \frac{x^2-1}{\sqrt{1+x}}dx = \int_1^{\sqrt{2}} [(u^2-1)^2 - 1]\, 2\,du$

$$\begin{cases} \text{let } u = \sqrt{1+x} \leadsto u^2 = 1+x \Rightarrow x = (u^2-1) \\ du = \frac{1}{2\sqrt{1+x}}dx \\ 2du = \frac{1}{\sqrt{1+x}}dx \end{cases}$$

$$\begin{cases} x = 0 \to u = 1 \\ x = 1 \to u = \sqrt{2} \end{cases}$$

$= 2\int_1^{\sqrt{2}} [u^4 + 1 - 2u^2 - 1]\, du$

$= 2\int_1^{\sqrt{2}} (u^4 - 2u^2)\, du$

$= 2\left[\frac{u^5}{5} - \frac{2u^3}{3}\right]_1^{\sqrt{2}}$

$= 2\left[\left(\frac{(\sqrt{2})^5}{5} - \frac{2}{3}(\sqrt{2})^3\right) - \left(\frac{1}{5} - \frac{2}{3}\right)\right]$

$= 2 \cdot \frac{4\sqrt{2}}{5} - \frac{4}{3} \cdot 2\sqrt{2} - \frac{2}{5} + \frac{4}{3} = \frac{8\sqrt{2}}{5} - \frac{8\sqrt{2}}{3} - \frac{2}{5} + \frac{4}{3} = \frac{14 - 16\sqrt{2}}{15}$

[4] 5. $\int \frac{x+2}{x^2+4}dx = \int \left(\frac{x}{x^2+4} + \frac{2}{x^2+4}\right)dx$

$$\begin{cases} \text{let } u = x^2+4 \\ du = 2x\,dx \leadsto \frac{du}{2} = x\,dx \end{cases}$$

$= \int \frac{x}{x^2+4}dx + 2\int \frac{1}{x^2+4}dx$

can also skip this

$$\begin{cases} \text{let } u = \frac{x}{2} \\ du = \frac{1}{2}dx \leadsto 2du = dx \end{cases}$$

$= \frac{1}{2}\int \frac{1}{u}\,du + 2\int \frac{1}{4\left(\frac{x^2}{4}+1\right)}dx$

$= \frac{1}{2}\ln|u| + \frac{1}{2}\int \frac{1}{\left(\frac{x}{2}\right)^2 + 1}dx$

$= \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\int \frac{1}{u^2+1}\,2du$

$= \frac{1}{2}\ln(x^2+4) + \tan^{-1}(u) + C = \frac{1}{2}\ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C$

[4] 6. $\int \sqrt{1+\sqrt{x}}dx$

$$\begin{cases} \text{let } u = 1+\sqrt{x} \leadsto \sqrt{x} = u-1 \\ du = \frac{1}{2\sqrt{x}}dx \leadsto 2\sqrt{x}\,du = dx \\ \qquad 2(u-1)\,du = dx \end{cases}$$

$= \int \sqrt{u} \cdot 2(u-1)\,du$

$= 2\int u^{1/2}(u-1)\,du$

$= 2\int \left(u^{3/2} - u^{1/2}\right)du$

$= 2\frac{u^{5/2}}{5/2} - 2\frac{u^{3/2}}{3/2} + C$

$= \frac{4}{5}(1+\sqrt{x})^{5/2} - \frac{4}{3}(1+\sqrt{x})^{3/2} + C$

v10.AB