Math 1210: Worksheet #10 **Fall 2017**

Name: **Solution**	A#:	Section:

[3] 1. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin(2x)}{\sin^2(x)+1}dx$

$u = \sin^2(x) + 1$

$du = 2\sin(x)\cos(x)\,dx = \sin(2x)\,dx$

$x = \frac{\pi}{4} \Rightarrow u = \sin^2(\frac{\pi}{4}) + 1 = \frac{3}{2}$

$x = \frac{\pi}{6} \Rightarrow u = \sin^2(\frac{\pi}{6}) + 1 = \frac{5}{4}$

$$= \int_{\frac{5}{4}}^{\frac{3}{2}} \frac{1}{u}\,du$$

$$= \ln u \Big|_{\frac{5}{4}}^{\frac{3}{2}} = \ln\frac{3}{2} - \ln\frac{5}{4} = \ln(\frac{6}{5})$$

[3] 2. $\int_0^{\ln(2)} \frac{e^{2x}}{e^{2x}+1}dx$

$u = e^{2x} + 1$

$du = 2e^{2x}\,dx$

$x = \ln(2) \Rightarrow u = e^{2\ln(2)} + 1 = 5$

$x = 0 \Rightarrow u = e^0 + 1 = 2$

$$= \int_2^5 \frac{1}{u}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\int_2^5 \frac{1}{u}\,du = \frac{1}{2}\ln u\Big|_2^5 = \frac{1}{2}(\ln 5 - \ln 2) = \frac{1}{2}\ln(\frac{5}{2})$$

[3] 3. $\int_0^{\frac{1}{2}\ln(3)} \frac{e^{x}}{e^{2x}+1}dx$

$u = e^x$

$du = e^x\,dx$

$x = \frac{1}{2}\ln(3) \Rightarrow u = e^{\ln(\sqrt{3})} = \sqrt{3}$

$x = 0 \Rightarrow u = e^0 = 1$

$$= \int_1^{\sqrt{3}} \frac{1}{u^2+1}\,du$$

$$= \tan^{-1}(u)\Big|_1^{\sqrt{3}} = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

[3] 4. $\int_0^1 \frac{x^2-1}{\sqrt{1+x}}dx$

$u = 1+x \longrightarrow x = u-1 \longrightarrow x^2 = (u-1)^2$

$du = dx$

$x = 1 \implies u = 1+x = 2$

$x = 0 \implies u = 1+x = 1$

$$= \int_1^2 \frac{(u-1)^2 - 1}{\sqrt{u}}\, du$$

$$= \int_1^2 u^{-\frac{1}{2}}(u^2 - 2u)\,du = \int_1^2 (u^{\frac{3}{2}} - 2u^{\frac{1}{2}})\,du = \left(\frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}}\right)\Big|_1^2$$

$$= \frac{2}{5}(2)^{\frac{5}{2}} - \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(1) + \frac{4}{3}(1) = \frac{8}{5}\sqrt{2} - \frac{8}{3}\sqrt{2} - \frac{2}{5} + \frac{4}{3}$$

[4] 5. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx = \int \frac{e^x}{e^x + e^{-x}}dx - \int \frac{e^{-x}}{e^x + e^{-x}}dx = \frac{1}{2}\ln(e^{2x}+1) + \frac{1}{2}\ln(1+e^{-2x}) + C$

$u = e^x$ $\quad v = u^2 + 1$

$du = e^x\,dx$ $\quad dv = 2u\,du$

$$\int \frac{e^x}{e^x + e^{-x}}dx = \int \frac{1}{u + \frac{1}{u}}du$$

$$= \int \frac{u}{u^2+1}du = \frac{1}{2}\int \frac{1}{v}dv$$

$$= \frac{1}{2}\ln v = \frac{1}{2}\ln(e^{2x}+1)$$

$u = e^{-x}$

$du = -e^{-x}dx$

$$\int \frac{e^{-x}}{e^x + e^{-x}}dx = \int \frac{-1}{\frac{1}{u} + u}du$$

$$= \int \frac{-u}{1+u^2}du = \frac{-1}{2}\ln(1+u^2)$$

$$= \frac{-1}{2}\ln(1+e^{-2x})$$

Another way to solve question 5:

$u = e^x + e^{-x}$

$du = e^x - e^{-x}$

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx = \int \frac{1}{u}du = \ln(e^x + e^{-x}) + C$$

[4] 6. $\int \sqrt{1+\sqrt{x}}dx$

$u = 1 + \sqrt{x} \longrightarrow \sqrt{x} = u - 1$

$du = \frac{1}{2\sqrt{x}}dx \longrightarrow 2(u-1)du = dx$

$$= \int \sqrt{u}\,(2u-2)\,du$$

$$= \int (2u^{\frac{3}{2}} - 2u^{\frac{1}{2}})\,du = \frac{4}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}} = \frac{4}{5}(1+\sqrt{x})^{\frac{5}{2}} - \frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} + C$$