Saint Mary's University
Department of Mathematics
and Computing Science

Name: _____

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Math 1211: Introductory Calculus II

Midterm Test

March 3, 2016 (7:00–9:00pm)

Instructor: J. Irving

Instructions:

- *No electronic devices, or aids of any kind, are permitted.*
- *There are 6 pages plus this cover page. Check that your test paper is complete.*
- *There are a total of 75 points. The value of each question is indicated in the margin.*
- *Answer in the spaces provided, using backs of pages for additional space if necessary.*
- *Show all your work. Insufficient justification will result in a loss of points.*

Page	**Maximum**	**Your Score**
1	14	
2	16	
3	8	
4	12	
5	15	
6	10	
Total	**75**	

1. Evaluate the following. Simplify your answers as much as possible:

[3] (a) $\displaystyle\int \frac{e^{2x}}{\sqrt{1+e^{2x}}}\,dx$

[5] (b) $\displaystyle\int \cos^3 x\, \sin^3 x\, dx$

[6] (c) $\displaystyle\int x^2(\ln x)^2\, dx$

[8] (d) $\displaystyle\int \frac{x^2}{(x^2+4)^2}\,dx$ [*Hint:* Trigonometric substitution.]

[8] (e) $\displaystyle\int \frac{x+1}{(x^2+9)(x-1)}\,dx$

2. Determine whether the following improper integrals converge or diverge.

[3] (a) $\displaystyle\int_1^2 \frac{dt}{(1-t)^3}$

[3] (b) $\displaystyle\int_1^\infty e^{-2x}\,dx$

[2] (c) $\displaystyle\int_0^\infty \frac{x}{1+x^6}\,dx$ [*Hint:* Use comparison.]

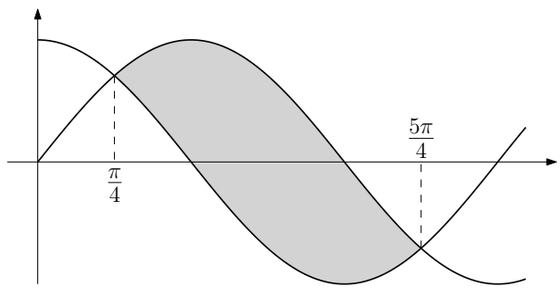
3. Let $\mathcal{R}$ be the region bounded between the curves $x = y^2 - 1$ and $x + y = 1$.

[3] (a) Sketch $\mathcal{R}$ and label all points of intersection of its bounding curves.

[4] (b) Find the area of $\mathcal{R}$.

[5] 4. The base of a solid is the region enclosed by the curve $y = x^4$ and the line $y = 1$. The cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

[15] 5. The shaded region $\mathcal{R}$ shown below is bounded between the curves $y = \cos x$ and $y = \sin x$.



Give expressions, in terms of definite integrals, for each of the following quantities.
Do not simplify or evaluate your expressions!

(a) The **volume** of the solid obtained by revolving $\mathcal{R}$ about the y axis.

(b) The **volume** of the solid obtained by revolving $\mathcal{R}$ about the line $x = 5$.

(c) The **volume** of the solid obtained by revolving $\mathcal{R}$ about the line $y = 2$.

(d) The **area** of $\mathcal{R}$.

(e) The **perimeter** of $\mathcal{R}$. [*Hint:* Use the formula for arc length.]

(f) The **surface area** of the solid obtained by revolving $\mathcal{R}$ about the line $y = -2$.

[10] 6. Consider the parametric curve $\mathcal{C}$ given by $(x, y) = (2\cos t, 1 + 2\sin t)$, for $0 \leq t \leq 2\pi$.

(a) Eliminate the parameter t to find the Cartesian equation of $\mathcal{C}$.

(b) Sketch $\mathcal{C}$, being sure to indicated the direction of travel.

(c) Find the equation of the tangent line to $\mathcal{C}$ at the point where $t = \dfrac{5\pi}{6}$.