Math 1211: Practice Worksheet #2 Winter 2016

Name: SOLUTIONS	A#:	Section:

1. Give the precise meanings of the following statements:

   (a) $\sum_{n=1}^{\infty} a_n$ converges

   $\lim_{n\to\infty} (a_1 + a_2 + a_3 + \cdots + a_n)$ exists

   (b) $\sum_{n=1}^{\infty} a_n$ converges absolutely

   $\sum_{n=1}^{\infty} |a_n|$ converges

   (c) $\sum_{n=1}^{\infty} a_n$ converges conditionally

   $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ diverges

2. Determine whether the following series converge absolutely, converge conditionally, or diverge:

   (a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^3-1}}$

   Note that $\sum_{n=2}^{\infty} \left|\frac{(-1)^n}{\sqrt{n^3-1}}\right| = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-1}}$

   Use limit comparison with $\sum \frac{1}{n^{3/2}}$ to see that

   $\sum \frac{1}{\sqrt{n^3-1}}$ converges. Therefore $\sum \frac{(-1)^n}{\sqrt{n^3-1}}$ $\boxed{\text{converges absolutely}}$

   I'm skipping the work here, but you should do it on the exam!

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \ln n}$

   The sequence $\frac{1}{n-\ln n}$ is eventually decreasing, since

   $\frac{d}{dx}\left(\frac{1}{x-\ln x}\right) = -\frac{(1-1/x)}{(x-\ln x)^2} < 0$ for $x > 1$. Since $\lim_{n\to\infty} \frac{1}{n-\ln n} = 0$,

   $\sum \frac{(-1)^n}{n-\ln n}$ converges by Leibniz Test.

   But $\sum \left|\frac{(-1)^n}{n-\ln n}\right| = \sum \frac{1}{n-\ln n}$ diverges, since

   $\frac{1}{n-\ln n} > \frac{1}{n}$ and $\sum \frac{1}{n}$ diverges.

   Therefore $\sum \frac{(-1)^n}{n-\ln n}$ $\boxed{\text{converges conditionally}}$

3. Determine the **radius** and **interval** of convergence of the following power series:

(a) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{5^n}$

Apply Ratio Test: $\rho = \lim_{n\to\infty} \left| \frac{(n+1)^2 x^{n+1}}{5^{n+1}} \cdot \frac{5^n}{n^2 x^n} \right|$

$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{5} \cdot |x|$

$= \frac{1}{5}|x|$

So the series converges absolutely when $\frac{1}{5}|x| < 1$, that is, $|x| < 5$.

Radius of convergence is $\boxed{R = 5}$

At $x = 5$ and $x = -5$, get $\sum n^2$ and $\sum (-1)^n n^2$ respectively, both of which diverge since their terms don't go to 0.

Therefore the interval of convergence is $\boxed{(-5, 5)}$.

(b) $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$

Apply Ratio Test: $\rho = \lim_{n\to\infty} \left| \frac{3^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n} \right| = \lim_{n\to\infty} \frac{3|x|}{n+1} = 0$

So the radius of convergence is $\boxed{R = \infty}$.

Therefore the interval of convergence is $\boxed{(-\infty, \infty)}$.

(c) $\sum_{n=1}^{\infty} \frac{2^n}{n}(x-1)^n$

Apply Ratio Test: $\rho = \lim_{n\to\infty} \left| \frac{2^{n+1}(x-1)^{n+1}}{n+1} \cdot \frac{n}{2^n (x-1)^n} \right|$

$= \lim_{n\to\infty} 2|x-1| \cdot \frac{n}{n+1}$

$= 2|x-1|$.

So we have absolute convergence when $2|x-1| < 1$, which is equivalent to $|x-1| < \frac{1}{2}$ or $x \in (\frac{1}{2}, \frac{3}{2})$.

Thus the radius of convergence is $\boxed{R = 1/2}$

At $x = \frac{3}{2}$ get $\sum \frac{1}{n}$, which diverges (harmonic series)

At $x = \frac{1}{2}$ get $\sum \frac{(-1)^n}{n}$, which converges by Leibniz test

So the interval of convergence is $\boxed{[\frac{1}{2}, \frac{3}{2})}$