Name: SOLUTIONS	A#:	Section:

1. Define precisely what it means to say that $\sum_{n=1}^{\infty} a_n = S$.

$$\lim_{N\to\infty} \underbrace{(a_1+a_2+a_3+\cdots+a_N)}_{N^{th}\text{ partial sum}} = S$$

2. Determine whether each of the following series converges or diverges. If a series converges, then find its value.

(a) $\sum_{n=0}^{\infty} 5e^{-n}$

Geometric with $r=\frac{1}{e}<1$ and $c=5$.

$\boxed{\text{CONVERGES to } \frac{5}{1-\frac{1}{e}} = \frac{5e}{e-1}}$

(b) $\sum_{n=1}^{\infty} \frac{4n+1}{5n-1}$

$\boxed{\text{DIVERGES}}$ since $\lim_{n\to\infty} \frac{4n+1}{5n+1} = \lim_{n\to\infty} \frac{4+1/n}{5+1/n} = \frac{4}{5} \neq 0$.

(c) $\sum_{n=0}^{\infty} \frac{2^{3n+1}}{3^{2n-1}} = \sum_{n=0}^{\infty} \frac{2}{1/3}\cdot\frac{(2^3)^n}{(3^2)^n} = \sum_{n=0}^{\infty} 6\cdot\left(\frac{8}{9}\right)^n$

Geometric with $c=6$ and $r=8/9$.

$\boxed{\text{CONVERGES to } \frac{6}{1-8/9} = 54}$

(d) $\sum_{n=1}^{\infty} \frac{2^n}{n^4}$

$\boxed{\text{DIVERGES}}$ since $\lim_{n\to\infty} \frac{2^n}{n^4} = \infty \neq 0$.

(e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n}} = \sum_{n=1}^{\infty} \left(\frac{-1}{4}\right)^n$

Geometric with $r=-\frac{1}{4}$ and $c=-\frac{1}{4}$

$\boxed{\text{CONVERGES to } \frac{-1/4}{1-(-1/4)} = -\frac{1}{5}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - 4^n}{3^n}$

$\boxed{\text{DIVERGES}}$ since $\displaystyle\lim_{n\to\infty} \frac{2^n-4^n}{3^n} = \lim_{n\to\infty}\left(\left(\frac{2}{3}\right)^n - \left(\frac{4}{3}\right)^n\right)$ $\quad (\left(\frac{2}{3}\right)^n \to 0,\ \left(\frac{4}{3}\right)^n \to \infty)$

$= -\infty$

$\neq 0$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n - 4^n}{5^n}$

$$= \sum_{n=1}^{\infty}\left(\left(\frac{3}{5}\right)^n - \left(\frac{4}{5}\right)^n\right)$$

$$= \sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n - \sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n \leftarrow$$ (since both series converge, as they are geometric with $r = 3/5$ and $r = 4/5$, respectively)

$$= \frac{3/5}{1-3/5} - \frac{4/5}{1-4/5}$$

$$= \frac{3}{2} - 4 = -\frac{5}{2}$$

$\boxed{\therefore \text{ CONVERGES to } -\frac{5}{2}}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

$$= \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$$= \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

The N^{th} partial sum is:

$$S_N = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{4}\right) + \cdots + \left(\cancel{\frac{1}{N}} - \frac{1}{N+1}\right)$$

$$= 1 - \frac{1}{N+1}$$

So $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n} = \lim_{N\to\infty} S_N = \lim_{N\to\infty}\left(1 - \frac{1}{N+1}\right) = 1$

(That is, $\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n} \text{ CONVERGES to } 1}$)