Math 1211: Quiz #2 12 Winter 2016

Name: Key	A#:	Section:

1. $\int x^2 e^{2x}\,dx$ let $u = x^2$, $du = 2x\,dx$; $dv = e^{2x}\,dx$, $v = \frac{1}{2}e^{2x}$ 4

$= \frac{x^2}{2}e^{2x} - \int \frac{1}{2}e^{2x}\cdot 2x\,dx$

$= \frac{x^2}{2}e^{2x} - \int x e^{2x}\,dx$ let $u = x$, $du = dx$; $dv = e^{2x}\,dx$, $v = \frac{1}{2}e^{2x}$

$= \frac{x^2}{2}e^{2x} - \left[\frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx\right]$

$= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{2}\cdot\frac{1}{2}e^{2x} + C = \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$

2. $\int \cos^{-1}x\,dx$ let $u = \cos^{-1}x$, $du = \frac{-1}{\sqrt{1-x^2}}$; $dv = dx$, $v = x$

$= x\cos^{-1}x - \int \frac{-x}{\sqrt{1-x^2}}\,dx$

$= x\cos^{-1}x + \int \frac{x}{\sqrt{1-x^2}}\,dx$ let $w = 1-x^2$, $dw = -2x\,dx$, $-\frac{dw}{2} = x\,dx$ 4

$= x\cos^{-1}x - \frac{1}{2}\int \frac{1}{\sqrt{w}}\,dw$

$= x\cos^{-1}x - \frac{1}{2}\frac{w^{1/2}}{1/2} + C = x\cos^{-1}x - \sqrt{w} + C = x\cos^{-1}x - \sqrt{1-x^2} + C$

3. $\int_0^{\pi} \sin^3 x\,dx$

$\int \sin^3 x\,dx = \int \sin^2 x \sin x\,dx$ let $u = \cos x$, $du = -\sin x\,dx$

$= \int (1-\cos^2 x)\sin x\,dx$

$= -\int (1-u^2)\,du$

$= \int (u^2 - 1)\,du$

$= \frac{u^3}{3} - u + C$ 4

$= \frac{\cos^3 x}{3} - \cos x + C$

$\therefore \int_0^{\pi} \sin^3 x\,dx = \left[\frac{\cos^3 x}{3} - \cos x\right]_0^{\pi}$

$= \left(\frac{\cos^3\pi}{3} - \cos\pi\right) - \left(\frac{\cos 0}{3} - \cos 0\right)$

$= \frac{-1}{3} - (-1) - \frac{1}{3} + 1$

$= \frac{-2}{3} + 2$

$= \frac{4}{3}$

v2.1

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1. $\int x\sec^2 x\,dx$ let $u = x$ ✓ $dv = \sec^2 x\,dx$

$du = dx$ $v = \tan x$

$$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx ✓$$

$$= x\tan x - \int \frac{\sin x}{\cos x}\,dx$$

let $w = \cos x$
$dw = -\sin x\,dx$
$-dw = \sin x\,dx$

4

$$= x\tan x - \int \frac{1}{w}(-dw) ✓$$

$$= x\tan x + \int \frac{1}{w}\,dw$$

$$= x\tan x + \ln|w| + C = x\tan x + \ln|\cos x| + C ✓$$

2. $\int \sin^{-1} x\,dx$ let $u = \sin^{-1} x$ ✓ $dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}$ $v = x$

$$\int \sin^{-1} x\,dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\,dx ✓$$

let $w = 1-x^2$
$dw = -2x\,dx$
$-\frac{dw}{2} = x\,dx$

4

$$= x\sin^{-1} x - \int \frac{1}{\sqrt{w}}\cdot\left(-\frac{dw}{2}\right)$$

$$= x\sin^{-1} x + \frac{1}{2}\int w^{-1/2}\,dw ✓$$

$$= x\sin^{-1} x + \frac{1}{2}\frac{w^{1/2}}{1/2} + C = x\sin^{-1} x + \sqrt{w} + C ✓$$

$$= x\sin^{-1} x + \sqrt{1-x^2} + C$$

3. $\int_0^{\pi/2} \cos^3 x\,dx$ Power of cosine is odd

$$\int \cos^3 x\,dx = \int \cos^2 x\cos x\,dx ✓$$

let $u = \sin x$
$du = \cos x\,dx$

$$= \int (1-\sin^2 x)\cos x\,dx$$

$$= \int (1-u^2)\,du ✓$$

$$= u - \frac{u^3}{3} + C ✓$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$

$$\therefore \int_0^{\pi/2} \cos^3 x\,dx = \left[\sin x - \frac{\sin^3 x}{3}\right]_0^{\pi/2}$$

$$= \sin\frac{\pi}{2} - \frac{\sin^3(\pi/2)}{3} - \sin 0 + \frac{\sin^3 0}{3}$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3} ✓$$

v2.2

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Name: SOLUTIONS	A#:	Section:

1. $\int x \cos 2x \, dx$

PARTS: $u = x$ $\quad dv = \cos 2x \, dx$
$du = dx$ $\quad v = \frac{1}{2}\sin 2x$

$= \frac{1}{2} x \sin 2x - \frac{1}{2}\int \sin 2x \, dx$

$= \frac{1}{2} x \sin 2x + \frac{1}{4}\cos 2x + C$

2. $\int \ln x \, dx$

PARTS: $u = \ln x$ $\quad dv = dx$
$du = \frac{1}{x} dx$ $\quad v = x$

$= x \ln x - \int 1 \, dx$

$= x \ln x - x + C$

3. $\int_0^{\pi/2} \cos^3 x \sin^3 x \, dx$

$= \int_0^{\pi/2} \cos^2 x \sin^3 x \cos x \, dx$

$u = \sin x, \quad du = \cos x$
$u(0) = 0, \quad u(\pi/2) = 1$

$= \int_0^1 (1-u^2) u^3 \, du$

$= \int_0^1 (u^3 - u^5) \, du$

$= \frac{1}{4}u^4 - \frac{1}{6}u^6 \Big]_0^1 = \frac{1}{4} - \frac{1}{6} = \boxed{\frac{1}{12}}$

v2.3