Math 1211: Quiz #3 Winter 2016

| Name: Key | A#: | Section: |
|---|---|---|

1. $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

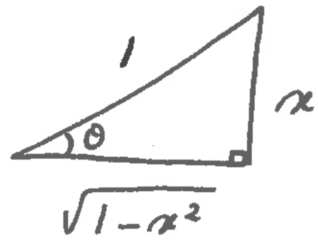
let $x = \sin\theta$
$dx = \cos\theta\, d\theta$

$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta}$
$= \sqrt{\cos^2\theta}$
$= \cos\theta$

$$= \int \frac{\sin^2\theta}{\cos\theta}\cos\theta\, d\theta$$

$$= \int \sin^2\theta\, d\theta$$

$$= \int \frac{1}{2}(1-\cos 2\theta)\, d\theta$$




$\cos\theta = \sqrt{1-x^2}$

$$= \frac{1}{2}\int 1\, d\theta - \frac{1}{2}\int \cos 2\theta\, d\theta$$

$$= \frac{1}{2}\theta - \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta + C$$

$$= \frac{1}{2}\theta - \frac{1}{4}2\sin\theta\cos\theta + C = \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta + C$$

$$= \frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2} + C$$

2. $\int \frac{2x+1}{(x+2)(x-3)}\,dx$

$$\frac{2x+1}{(x+2)(x-3)} = \frac{A}{(x+2)} + \frac{B}{(x-3)}$$

$$2x+1 = A(x-3) + B(x+2)$$
$$= Ax - 3A + Bx + 2B$$
$$= (A+B)x + (2B-3A)$$

$$\left.\begin{array}{l} A+B = 2 \\ 2B - 3A = 1 \end{array}\right\} \;(\times 3)\; \left.\begin{array}{l} 3A + 3B = 6 \\ -3A + 2B = 1 \end{array}\right\} \quad 5B = 7 \Rightarrow \boxed{B = \frac{7}{5}}$$

$$A = 2 - B \Rightarrow A = 2 - \frac{7}{5} \Rightarrow \boxed{A = \frac{3}{5}}$$

$$\int \frac{2x+1}{(x+2)(x+3)}\,dx = \int\left(\frac{3/5}{x+2} + \frac{7/5}{x-3}\right)dx$$

v3.1

$$= \frac{3}{5}\int \frac{1}{x+2}\,dx + \frac{7}{5}\int \frac{1}{x-3}\,dx$$

$$= \frac{3}{5}\ln|x+2| + \frac{7}{5}\ln|x-3| + C$$

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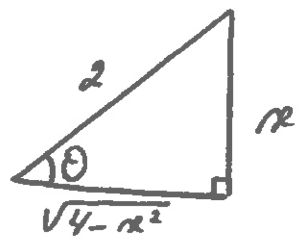
| Name: Key | A#: | Section: |
|---|---|---|

1. $\int \sqrt{4-x^2}\,dx$

Let $x = 2\sin\theta$
$dx = 2\cos\theta\, d\theta$

$$\sqrt{4-x^2} = \sqrt{4-(2\sin\theta)^2} = \sqrt{4-4\sin^2\theta} = 2\sqrt{1-\sin^2\theta} = 2\sqrt{\cos^2\theta} = 2\cos\theta$$

$$= \int 2\cos\theta \cdot 2\cos\theta\, d\theta$$
$$= 4\int \cos^2\theta\, d\theta$$
$$= 4\int \frac{1}{2}(1+\cos 2\theta)\, d\theta$$
$$= 2\int 1\, d\theta + 2\int \cos 2\theta\, d\theta$$
$$= 2\theta + 2\cdot\frac{1}{2}\sin 2\theta + C$$
$$= 2\theta + 2\sin\theta\cos\theta + C$$




$\sin\theta = \frac{x}{2}$

$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

$$= 2\sin^{-1}\left(\frac{x}{2}\right) + 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C = 2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{2} + C$$

2. $\int \frac{3x-4}{(x+1)(x-2)}\,dx = \int \left(\frac{7/3}{x+1} + \frac{2/3}{x-2}\right) dx$

$$\frac{3x-4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$
$$3x-4 = A(x-2) + B(x+1)$$
$$= Ax - 2A + Bx + B$$
$$= (A+B)x + (B-2A)$$

$A + B = 3 \Rightarrow B = 3 - A$

$B - 2A = -4 \Rightarrow (3-A) - 2A = -4$

$\therefore \boxed{A = \frac{7}{3}}$

and $\boxed{B = \frac{2}{3}}$

$$= \frac{7}{3}\int \frac{1}{x+1}\,dx + \frac{2}{3}\int \frac{1}{x-2}\,dx$$
$$= \frac{7}{3}\ln|x+1| + \frac{2}{3}\ln|x-2| + C$$

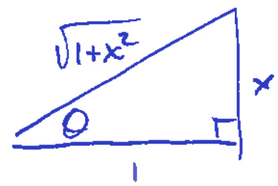
v3.2

**Math 1211: Quiz #3** **Winter 2016**

| Name: SOLUTIONS | A#: | Section: |
|---|---|---|

1. $\displaystyle\int \frac{dx}{(1+x^2)^2}$

Let $x = \tan\theta$, $dx = \sec^2\theta\, d\theta$, $1+x^2 = \sec^2\theta$




$$= \int \frac{\sec^2\theta}{(\sec^2\theta)^2}\, d\theta$$

$$= \int \frac{d\theta}{\sec^2\theta}$$

$$= \int \cos^2\theta\, d\theta$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$$

$$= \frac{1}{2}\tan^{-1}x + \frac{1}{2}\cdot\frac{x}{\sqrt{1+x^2}}\cdot\frac{1}{\sqrt{1+x^2}} + C$$

$$= \boxed{\frac{1}{2}\tan^{-1}x + \frac{x}{2(1+x^2)} + C}$$

2. $\displaystyle\int \frac{2-3x}{(x-1)(x+2)}\, dx$

Set $\displaystyle\frac{2-3x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \Rightarrow 2-3x = A(x+2) + B(x-1)$

Letting $x=1$ gives $-1 = 3A \Rightarrow A = -1/3$

Letting $x=-2$ gives $8 = -3B \Rightarrow B = -8/3$

So $\displaystyle\int \frac{2-3x}{(x-1)(x+2)}\, dx = \int \left(-\frac{1}{3}\cdot\frac{1}{x-1} - \frac{8}{3}\cdot\frac{1}{x+2}\right) dx$

$$= \boxed{-\frac{1}{3}\ln|x-1| - \frac{8}{3}\ln|x+2| + C}$$

v3.3