Math 1211: Quiz #4 Winter 2016

Name: Key	A#:	Section:

1. Evaluate the following improper integrals, or show that they diverge.

(a) $\int_1^\infty \frac{x}{1+x^2}\,dx = \lim_{R\to\infty} \int_1^R \frac{x}{1+x^2}\,dx$

$= \lim_{R\to\infty} \left[\frac{1}{2}\ln(1+x^2)\right]_1^R$

$= \lim_{R\to\infty} \left[\frac{1}{2}\ln(1+R^2) - \frac{1}{2}\ln 2\right]$

$= \frac{1}{2}\lim_{R\to\infty} \ln(1+R^2) - \frac{1}{2}\ln 2$

$= \infty$ Diverges.

$\begin{cases} \text{let } u = 1+x^2 \\ du = 2x\,dx \end{cases}$

$\int \frac{x}{1+x^2}\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|1+x^2| + C$

(b) $\int_{-6}^{2} \frac{dx}{\sqrt[3]{2-x}} = \lim_{R\to 2^-} \int_{-6}^{R} \frac{dx}{\sqrt[3]{2-x}}$

$= \lim_{R\to 2^-} \left[-\frac{3}{2}(2-x)^{2/3}\right]_{-6}^{R}$

$= -\frac{3}{2}\lim_{R\to 2^-} \left[(2-R)^{2/3} - 8^{2/3}\right]$

$= -\frac{3}{2}\lim_{R\to 2^-} (2-R)^{2/3} - \frac{3}{2}(-4)$

$= 0 + 6 = 6$ Converges.

$\int \frac{dx}{\sqrt[3]{2-x}}$ $\begin{cases} \text{let } u = 2-x \\ du = -dx \end{cases}$

$= -\int \frac{du}{u^{1/3}}$

$= -\int u^{-1/3}\,du$

$= -\frac{3}{2}u^{2/3} + C$

$= -\frac{3}{2}(2-x)^{2/3} + C$

2. Find the area bounded between the curves $y = 1 - x$ and $y = e^x$ over the interval $[0, 2]$.

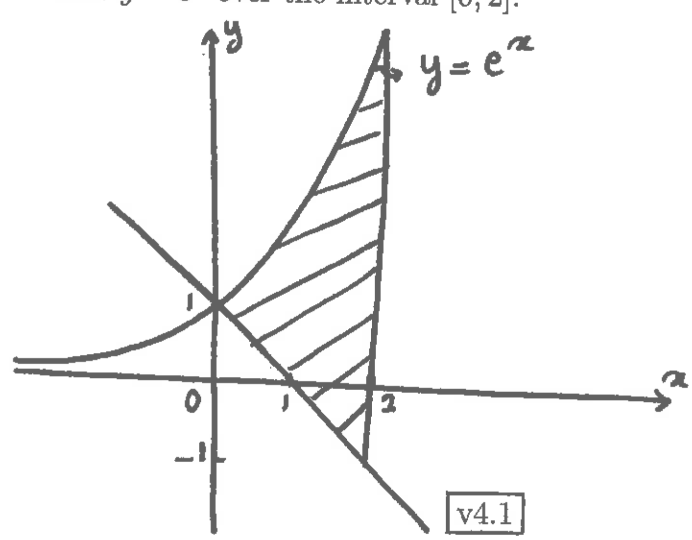
$A = \int_0^2 (y_T - y_B)\,dx$

$= \int_0^2 (e^x - (1-x))\,dx$

$= \int_0^2 (e^x - 1 + x)\,dx$

$= \left[e^x - x + \frac{x^2}{2}\right]_0^2$

$= \left(e^2 - 2 + \frac{4}{2}\right) - (e^0 - 0) = e^2 - 1$



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Math 1211: Quiz #4 | Winter 2016

Name: Key	A#:	Section:

1. Evaluate the following improper integrals, or show that they diverge.

(a) $\int_1^\infty e^{-2x}\,dx$

$= \lim_{R\to\infty} \int_1^R e^{-2x}\,dx$

$= \lim_{R\to\infty} \left[-\frac{1}{2}e^{-2x}\right]_1^R$

$= -\frac{1}{2}\lim_{R\to\infty}\left[e^{-2R} - e^{-2}\right]$

$= \frac{-1}{2}\lim_{R\to\infty}\frac{1}{e^{2R}} + \frac{1}{2e^2} = \frac{1}{2e^2}$ Converge.

$\int e^{-2x}\,dx$ $\left\{\begin{array}{l} \text{let } u=-2x \\ du=-2dx \\ \frac{du}{-2}=dx \end{array}\right.$

$= \frac{-1}{2}\int e^u\,du$

$= \frac{-1}{2}e^u + C = \frac{-1}{2}e^{-2x} + C$

(b) $\int_1^2 \frac{dx}{(2-x)^{3/2}}$

$= \lim_{R\to 2^-}\int_1^R \frac{dx}{(2-x)^{3/2}}$

$= \lim_{R\to 2^-}\left[\frac{2}{\sqrt{2-x}}\right]_1^R$

$= \lim_{R\to 2^-}\left[\frac{2}{\sqrt{2-R}} - \frac{2}{\sqrt{1}}\right]$

$= 2\lim_{R\to 2^-}\frac{1}{\sqrt{2-R}} - 2 = \infty$ Diverge.

$\int \frac{dx}{(2-x)^{3/2}}$ $\left\{\begin{array}{l} \text{let } u=2-x \\ du=-dx \\ -du=dx \end{array}\right.$

$= -\int \frac{1}{u^{3/2}}\,du$

$= -\frac{u^{-1/2}}{-1/2} + C$

$= \frac{2}{\sqrt{u}} + C$

$= \frac{2}{\sqrt{2-x}} + C$

2. Find the area bounded between the curves $y = 1+x$ and $y = \cos x$ over the interval $[0,\pi]$.

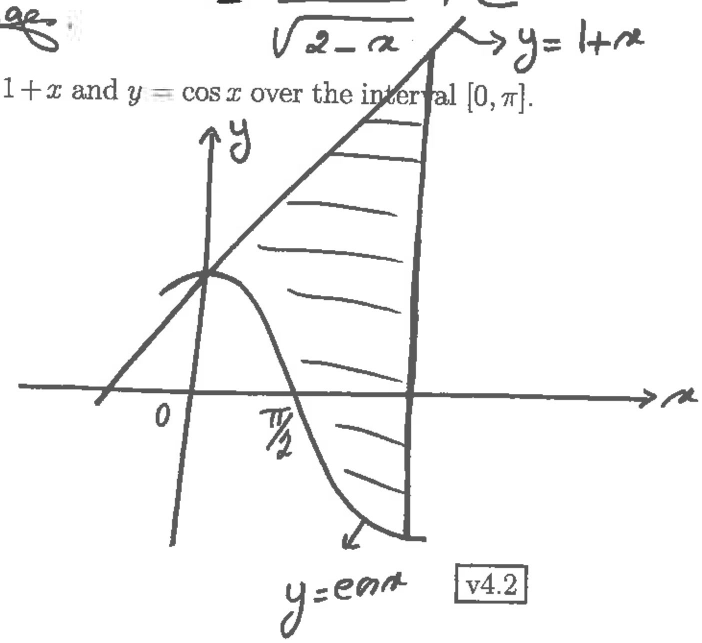
$A = \int_0^\pi (y_T - y_B)\,dx$

$= \int_0^\pi [(1+x) - \cos x]\,dx$

$= \left[x + \frac{x^2}{2} - \sin x\right]_0^\pi$

$= \left(\pi + \frac{\pi^2}{2} - \sin\pi\right) - (0)$

$= \pi + \frac{\pi^2}{2}$ or $\frac{1}{2}\pi(2+\pi)$



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Math 1211: Quiz #4 — Winter 2016

Name: SOLUTIONS	A#:	Section:

1. Evaluate the following improper integrals, or show that they diverge.

(a) $\displaystyle\int_{-\infty}^{1} e^{-2x}\,dx$

$$= \lim_{R\to-\infty} \int_R^1 e^{-2x}\,dx$$

$$= \lim_{R\to-\infty} -\frac{1}{2}e^{-2x}\Big]_R^1 = \lim_{R\to-\infty}\left(-\frac{1}{2}e^{-2} + \overbrace{\frac{1}{2}e^{-2R}}^{\to\infty \text{ as } R\to-\infty}\right) = \boxed{\infty}$$

(DIVERGES)

(b) $\displaystyle\int_{1}^{2} \frac{dx}{(2-x)^{2/3}}$

$$= \lim_{R\to2^-} \int_1^R \frac{dx}{(2-x)^{2/3}}$$

$$= \lim_{R\to2^-} -3(2-x)^{3/2}\Big]_1^R = \lim_{R\to2^-}\left(-3(2-R)^{3/2}+3\right) = \boxed{3}$$

(CONVERGES)

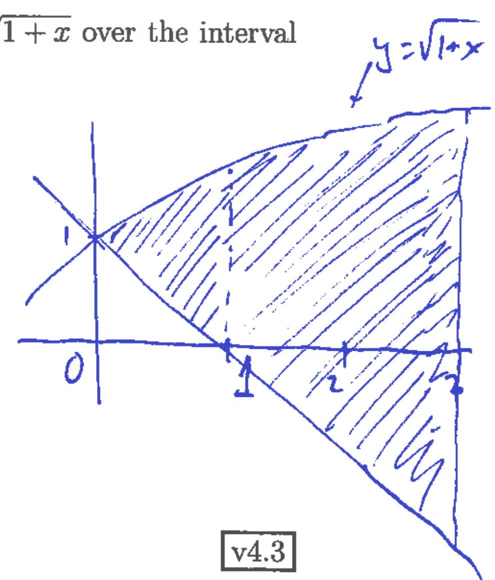
2. Find the area bounded between the curves $y = 1 - x$ and $y = \sqrt{1+x}$ over the interval $[0, 3]$.

$$\text{Area} = \int_0^3 \left(\sqrt{1+x} - (1-x)\right)dx$$

$$= \left(\frac{2}{3}(1+x)^{3/2} - x + \frac{1}{2}x^2\right)\Big|_0^3$$

$$= \left(\frac{16}{3} - 3 + \frac{9}{2}\right) - \frac{2}{3}$$

$$= \boxed{\frac{37}{6}}$$



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