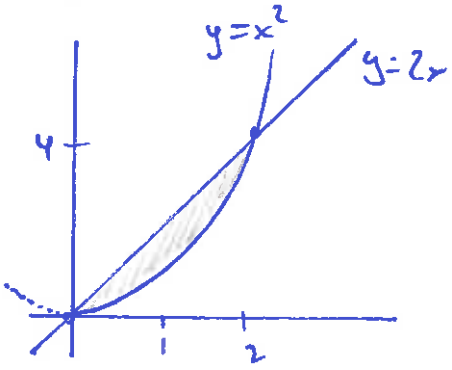
Math 1211: Quiz #5 **Winter 2016**

Name: SOLUTIONS	A#:	Section:

1. Let $\mathcal{R}$ be the region bounded between $y = 2x$ and $y = x^2$.

   (a) Sketch $\mathcal{R}$ and find its area.



$2x = x^2 \iff x(2-x) = 0$

$\iff x = 0$ or $x = 2$

$$\text{Area} = \int_0^2 (2x - x^2)\,dx$$

$$= x^2 - \frac{x^3}{3}\Big]_0^2$$

$$= 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$$

   (b) Find the volume of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

$$\text{Volume} = \pi \int_0^2 \left((2x)^2 - (x^2)^2\right) dx$$

$$= \pi \int_0^2 (4x^2 - x^4)\,dx$$

$$= \pi \cdot \left(\frac{4}{3}x^3 - \frac{1}{5}x^5\right)\Big|_{x=0}^{2}$$

$$= \pi\left(\frac{32}{3} - \frac{32}{5}\right)$$

$$= \boxed{\frac{64\pi}{15}}$$

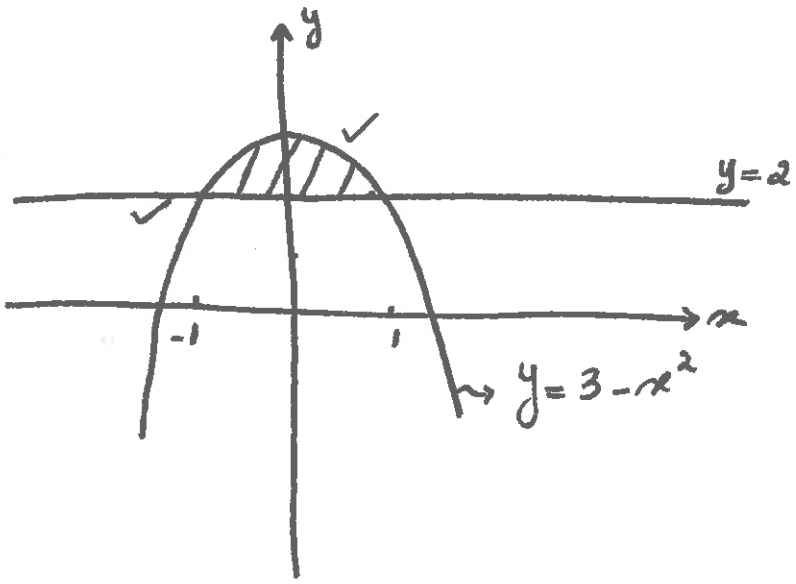
v5.1

Math 1211: Quiz #5 — 10 — Winter 2016

Name: Key	A#:	Section:

1. Let $\mathcal{R}$ be the region bounded between $y = 3 - x^2$ and $y = 2$.

(5) (a) Sketch $\mathcal{R}$ and find its area.



Points of intersection:

$3 - x^2 = 2 \Leftrightarrow x^2 = 1$

$\Leftrightarrow x = \pm 1$

$$\text{Area} = \int_{-1}^{1} (y_{Top} - y_B)\, dx$$

$$= \int_{-1}^{1} (3 - x^2 - 2)\, dx$$

$$= \int_{-1}^{1} (1 - x^2)\, dx$$

$$= \left[x - \frac{x^3}{3}\right]_{-1}^{1} = \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) = 2 - \frac{2}{3} = \boxed{\frac{4}{3}}$$

(5) (b) Find the volume of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

$R_i = 2$

$R_o = 3 - x^2$

$$V = \int_{-1}^{1} \pi (R_o^2 - R_i^2)\, dx$$

$$= \pi \int_{-1}^{1} ((3 - x^2)^2 - 2^2)\, dx$$

$$= \pi \int_{-1}^{1} (9 + x^4 - 6x^2 - 4)\, dx$$

$$= \pi \int_{-1}^{1} (x^4 - 6x^2 + 5)\, dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{6x^3}{3} + 5x\right]_{-1}^{1}$$

$$= \pi \left[\left(\frac{1}{5} - 2 + 5\right) - \left(\frac{-1}{5} + 2 - 5\right)\right] = \pi \left(\frac{2}{5} + 6\right) = \boxed{\frac{32\pi}{5}}$$

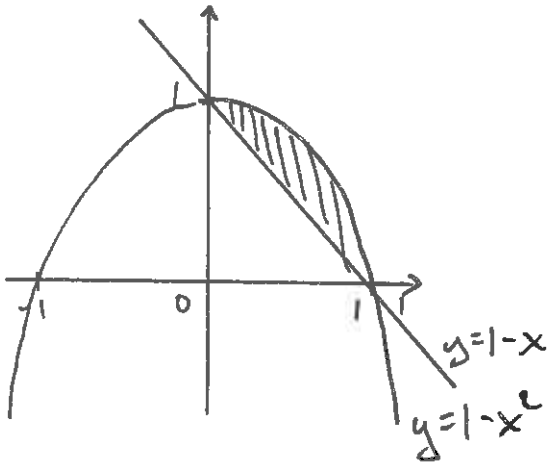
v5.2

Math 1211: Quiz #5 **Winter 2016**

Name: SOLUTION	A#:	Section:

1. Let $\mathcal{R}$ be the region bounded between $y = 1 - x^2$ and $y = 1 - x$.

   (a) Sketch $\mathcal{R}$ and find its area.



$$\text{Area} = \int_0^1 \left((1-x^2) - (1-x)\right) dx$$

$$= \int_0^1 (x - x^2)\, dx$$

$$= \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \boxed{\frac{1}{6}}$$

   (b) Find the volume of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

$$\text{Volume} = \pi \int_0^1 \left((1-x^2)^2 - (1-x)^2 \right) dx$$

$$= \pi \int_0^1 \left(1 - 2x^2 + x^4 - 1 + 2x - x^2 \right) dx$$

$$= \pi \int_0^1 \left(x^4 - 3x^2 + 2x \right) dx$$

$$= \pi \left(\frac{1}{5}x^5 - x^3 + x^2 \right) \Big|_0^1$$

$$= \boxed{\frac{\pi}{5}}$$

v5.3