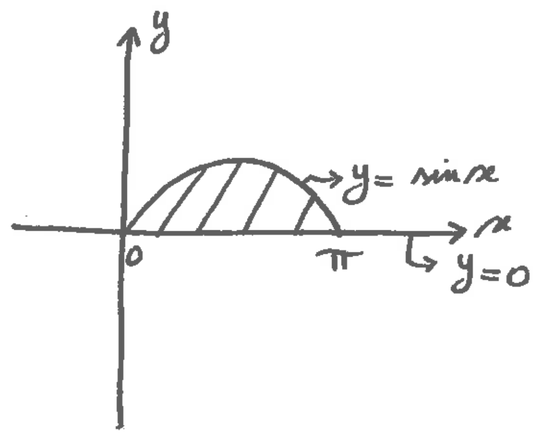
Math 1211: Quiz #6 **Winter 2016**

Name: Key	A#:	Section:

1. Find the volume of the solid obtained by revolving the region bounded by $y = \sin x$, $y = 0$ and $x = \pi$ about the y-axis.



Radius $= x$

height $= \sin x$

$$V = 2\pi \int_0^{\pi} x \sin x \, dx$$

$$= 2\pi \left(\Big[-x\cos x \Big]_0^{\pi} + \int_0^{\pi} \cos x \, dx \right)$$

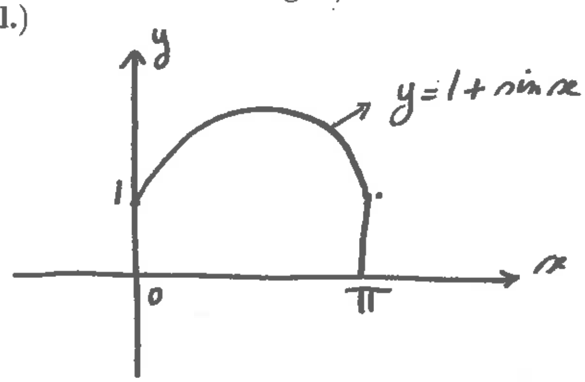
$$= 2\pi \left(-\pi \cos\pi + 0 + \sin x \Big]_0^{\pi} \right)$$

$$= 2\pi \left(-\pi(-1) + \sin\pi - \sin 0 \right)$$

$$= 2\pi^2$$

$$\begin{cases} \text{let } u = x & dv = \sin x \, dx \\ du = dx & v = -\cos x \end{cases}$$

2. Consider the surface obtained by revolving the graph of $y = 1 + \sin x$ between $x = 0$ and $x = \pi$ about the x-axis. Give an expression, in terms of a definite integral, for the area of this surface. (**Do not evaluate your integral.**)



$y = 1 + \sin x$

$y' = \cos x$

$$SA = 2\pi \int_0^{\pi} (1 + \sin x) \sqrt{1 + \cos^2 x} \, dx$$

v6.1

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Name: Key	A#:	Section:

1. Let $\mathcal{R}$ be the region between $y = \ln x$ and the x-axis over the interval $1 \leq x \leq e$. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis.

$$V = 2\pi \int_1^e x \ln x \, dx$$

let $u = \ln x$, $dv = x$; $du = \frac{1}{x} dx$, $v = \frac{x^2}{2}$

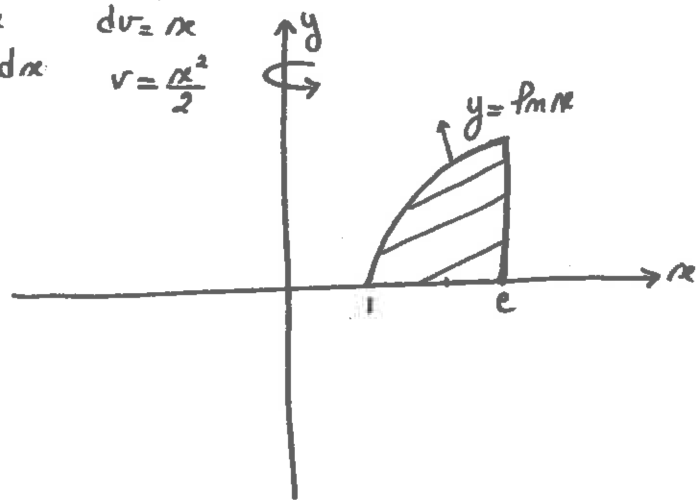
$$= 2\pi \left[\frac{x^2}{2} \ln x \right]_1^e - 2\pi \int_1^e \frac{x}{2} dx$$

$$= 2\pi \left[\frac{e^2}{2} \ln e - \frac{1}{2} \ln 1 \right] - \pi \left[\frac{x^2}{2} \right]_1^e$$

$$= 2\pi \left[\frac{e^2}{2} - 0 \right] - \pi \left[\frac{e^2}{2} - \frac{1}{2} \right]$$

$$= \pi e^2 - \frac{\pi e^2}{2} + \frac{\pi}{2}$$

$$= \frac{\pi e^2}{2} + \frac{\pi}{2} \text{ or } \frac{\pi}{2}(e^2 + 1)$$

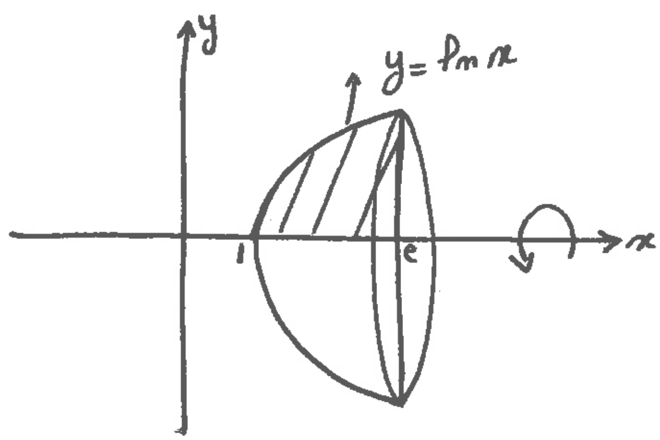


2. The graph of $y = \ln x$ between $x = 1$ and $x = e$ is revolved about the x-axis to create a surface $\mathcal{S}$. Give an expression, in terms of a definite integral, for the area of $\mathcal{S}$.
Do not evaluate your integral.

$$SA = 2\pi \int_1^e \ln x \sqrt{1 + \left(\frac{1}{x}\right)^2} \, dx$$

$y = \ln x$

$y' = \frac{1}{x}$

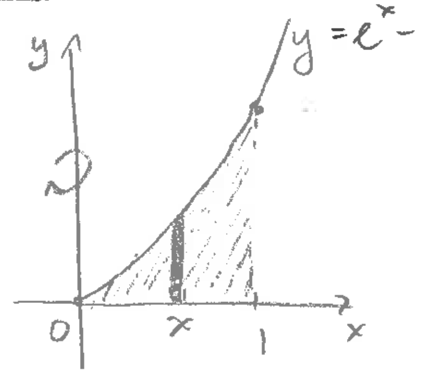


v6.2

Math 1211: Quiz #6 Winter 2016

Name: SOLUTIONS	A#:	Section:

1. Let $\mathcal{R}$ be the region between $y = e^x - 1$ and the x-axis over the interval $0 \le x \le 1$. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis.



$$V = 2\pi \int_0^1 x(e^x - 1)\,dx$$

$$= 2\pi \int_0^1 (xe^x - x)\,dx$$

$$= 2\pi \left(xe^x - e^x - \tfrac{1}{2}x^2\right)\Big]_0^1$$

$$= 2\pi\left(\left(e - e - \tfrac{1}{2}\right) - (0 - 1 - 0)\right)$$

$$= \boxed{\pi}$$

PARTS: $u = x$, $dv = e^x\,dx$, $du = dx$, $v = e^x$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$$

2. The graph of $y = e^x - 1$ between $x = 0$ and $x = 1$ is revolved about the x-axis to create a surface $\mathcal{S}$. Give an expression, in terms of a definite integral, for the area of $\mathcal{S}$.
Do not evaluate your integral.

$$\frac{dy}{dx} = e^x$$

$$\text{Area} = 2\pi \int_0^1 (e^x - 1)\sqrt{1 + (e^x)^2}\,dx$$

v6.3