

Math 1211: Quiz #6

Winter 2016

Name:

Ray

A#:

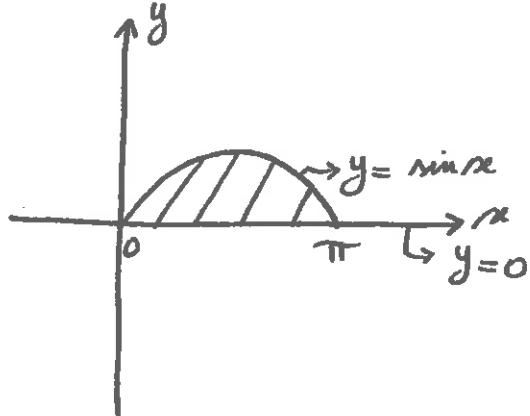
Section:

1. Find the volume of the solid obtained by revolving the region bounded by $y = \sin x$, $y = 0$ and $x = \pi$ about the y -axis.

$$\text{Radius} = x$$

$$\text{height} = \sin x$$

$$\begin{aligned} V &= 2\pi \int_0^{\pi} x \sin x \, dx \\ &= 2\pi \left(\left[-x \cos x \right]_0^{\pi} + \int_0^{\pi} \cos x \, dx \right) \\ &= 2\pi \left(-\pi \cos \pi + 0 + \sin x \Big|_0^{\pi} \right) \\ &= 2\pi \left(-\pi(-1) + \sin \pi - \sin 0 \right) \\ &= 2\pi^2 \end{aligned}$$



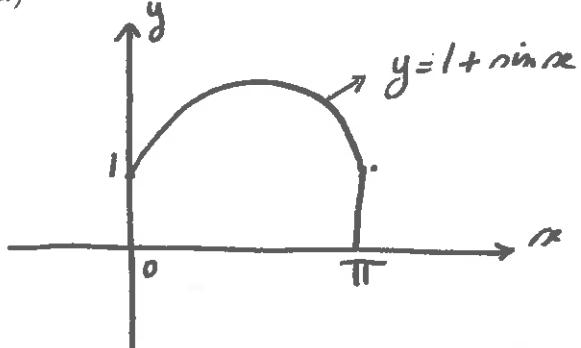
$$\begin{cases} \text{let } u = x & du = \sin x \, dx \\ du = dx & v = -\cos x \end{cases}$$

2. Consider the surface obtained by revolving the graph of $y = 1 + \sin x$ between $x = 0$ and $x = \pi$ about the x -axis. Give an expression, in terms of a definite integral, for the area of this surface. (Do not evaluate your integral.)

$$y = 1 + \sin x$$

$$y' = \cos x$$

$$SA = 2\pi \int_0^{\pi} (1 + \sin x) \sqrt{1 + \cos^2 x} \, dx$$



Name: *Key*

A#:

Section:

1. Let \mathcal{R} be the region between $y = \ln x$ and the x -axis over the interval $1 \leq x \leq e$. Find the volume of the solid obtained by revolving \mathcal{R} about the y -axis.

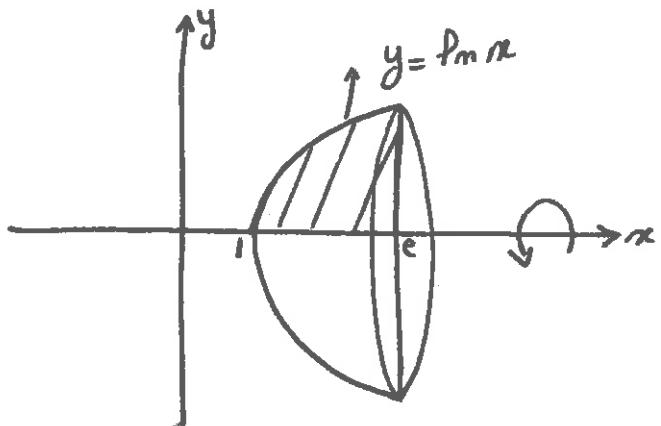
$$\begin{aligned}
 V &= 2\pi \int_1^e x P_m x \, dx && \left\{ \begin{array}{l} u = P_m x \\ du = \frac{1}{x} dx \\ v = \frac{x^2}{2} \end{array} \right. \\
 &= 2\pi \left[\frac{x^2}{2} P_m x \right]_1^e - 2\pi \int_1^e \frac{x}{2} \, dx \\
 &= 2\pi \left[\frac{e^2}{2} P_m e - \frac{1}{2} P_m 1 \right] - \pi \left[\frac{x^2}{2} \right]_1^e \\
 &= 2\pi \left[\frac{e^2}{2} - 0 \right] - \pi \left[\frac{e^2}{2} - \frac{1}{2} \right] \\
 &= \pi e^2 - \frac{\pi e^2}{2} + \frac{\pi}{2} \\
 &= \frac{\pi e^2}{2} + \frac{\pi}{2} \quad \cong \quad \frac{\pi}{2} (e^2 + 1)
 \end{aligned}$$

The graph shows the region R in the first quadrant, bounded above by the curve y = ln x, below by the x-axis, and on the left and right by the vertical lines x = 1 and x = e respectively. The region is shaded with diagonal lines.

2. The graph of $y = \ln x$ between $x = 1$ and $x = e$ is revolved about the x -axis to create a surface S . Give an expression, in terms of a definite integral, for the area of S . Do not evaluate your integral.

$$SA = 2\pi \int_1^e P_m x \sqrt{1 + \left(\frac{1}{x}\right)^2} \, dx$$

$$\begin{aligned}
 y &= P_m x \\
 y' &= \frac{1}{x}
 \end{aligned}$$



Name: SOLUTIONS

A#:

Section:

1. Let \mathcal{R} be the region between $y = e^x - 1$ and the x -axis over the interval $0 \leq x \leq 1$. Find the volume of the solid obtained by revolving \mathcal{R} about the y -axis.

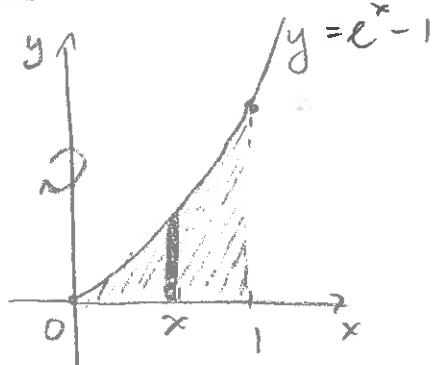
$$V = 2\pi \int_0^1 x(e^x - 1) dx$$

$$= 2\pi \int_0^1 (xe^x - x) dx$$

$$= 2\pi \left[xe^x - e^x - \frac{1}{2}x^2 \right]_0^1$$

$$= 2\pi \left((e - e - \frac{1}{2}) - (0 - 1 - 0) \right)$$

$$= \boxed{\pi}$$



PARTS: $u = x \quad du = e^x dx$
 $du = dx \quad v = e^x$

$$\int xe^x dx = xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

2. The graph of $y = e^x - 1$ between $x = 0$ and $x = 1$ is revolved about the x -axis to create a surface S . Give an expression, in terms of a definite integral, for the area of S . Do not evaluate your integral.

$$\frac{dy}{dx} = e^x$$

$$\text{Area} = 2\pi \int_0^1 (e^x - 1) \sqrt{1 + (e^x)^2} dx$$