Math 1211: Quiz #8 **Winter 2016**

Name: Key	A#:	Section:

1. Find the 3rd order (degree) Maclaurin polynomial of $f(x) = \ln(1+x)$.

$f(x) = \ln(1+x)$ $\qquad f(0) = 0$

$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$ $\qquad f'(0) = 1$

$f''(x) = -(1+x)^{-2}$ $\qquad f''(0) = -1$

$f'''(x) = 2(1+x)^{-3}$ $\qquad f'''(0) = 2$

$T_3(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$

$= 0 + 1(x) - \frac{1}{2}x^2 + \frac{2}{6}x^3$

$= x - \frac{x^2}{2} + \frac{x^3}{3}$

2. Find the 2nd order (degree) Taylor polynomial of $f(x) = \sqrt[3]{x}$ centred at 1.

$f(x) = x^{1/3}$ $\qquad f(1) = 1$

$f'(x) = \frac{1}{3}x^{-2/3}$ $\qquad f'(1) = \frac{1}{3}$

$f''(x) = -\frac{2}{9}x^{-5/3}$ $\qquad f''(1) = -\frac{2}{9}$

$T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$

$= 1 + \frac{1}{3}(x-1) - \frac{2}{9} \cdot \frac{(x-1)^2}{2}$

$= 1 + \frac{1}{3}(x-1) - \frac{1}{9}(x-1)^2$

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Name: Key	A#:	Section:

1. Find the 3rd order (degree) Maclaurin polynomial of $f(x) = (1+x)^{3/2}$.

$$f(x) = (1+x)^{3/2} \qquad f(0) = 1$$

$$f'(x) = \frac{3}{2}(1+x)^{1/2} \qquad f'(0) = \frac{3}{2}$$

$$f''(x) = \frac{3}{4}(1+x)^{-1/2} \qquad f''(0) = \frac{3}{4}$$

$$f'''(x) = \frac{-3}{8}(1+x)^{-3/2} \qquad f'''(0) = \frac{-3}{8}$$

$$T_3(x) = 1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3$$

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2. Find the 2nd order (degree) Taylor polynomial of $f(x) = \ln x$ centred at 1.

$$f(x) = \ln x \qquad f(1) = \ln 1 = 0$$

$$f'(x) = \frac{1}{x} \qquad f'(1) = 1$$

$$f''(x) = -\frac{1}{x^2} \qquad f''(1) = -1$$

$$T_2(x) = 0 + (x-1) - \frac{1}{2}(x-1)^2$$

$$= (x-1) - \frac{1}{2}(x-1)^2$$

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v8.2

Math 1211: Quiz #8 — Winter 2016

Name: SOLUTIONS	A#:	Section:

1. Find the 3rd order (degree) Maclaurin polynomial of $f(x) = e^x - \sin x$.

$$f(x) = e^x - \sin x \Rightarrow f(0) = 1$$
$$f'(x) = e^x - \cos x \Rightarrow f'(0) = 0$$
$$f''(x) = e^x + \sin x \Rightarrow f''(0) = 1$$
$$f'''(x) = e^x + \cos x \Rightarrow f'''(0) = 2$$

So $M_3(x) = 1 + 0x + \frac{1}{2!}x^2 + \frac{2}{3!}x^3$

$$= \boxed{1 + \frac{x^2}{2} + \frac{x^3}{3}}$$

2. Find the 2nd order (degree) Taylor polynomial of $f(x) = (x+1)^{3/2}$ centred at 3.

$$f(x) = (x+1)^{3/2} \Rightarrow f(3) = 8$$
$$f'(x) = \frac{3}{2}(x+1)^{1/2} \Rightarrow f'(3) = 3$$
$$f''(x) = \frac{3}{4}(x+1)^{-1/2} \Rightarrow f''(3) = \frac{3}{8}$$

So $T_2(x) = 8 + 3(x-3) + \frac{3/8}{2!}(x-3)^2$

$$= \boxed{8 + 3(x-3) + \frac{3}{16}(x-3)^2}$$