Math 1211: Quiz #9 Winter 2016

Name: Key	A#:	Section:

1. Define precisely what is meant when we say that $\sum_{n=1}^{\infty} a_n$ converges to S.

An infinite series $\sum_{n=1}^{\infty} a_n$ converges to the sum S if its partial sums converge to S:

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$$\lim_{N\to\infty} S_N = S \quad \text{where } S_N = a_1 + a_2 + \cdots + a_N$$

2. Determine whether the following series converge or diverge:

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$

4

$$\sqrt{n} + 2^n \geq 2^n$$

$$\frac{1}{\sqrt{n}+2^n} \leq \frac{1}{2^n}$$

$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \rightsquigarrow$ geometric series with $r = \frac{1}{2} < 1$ $\therefore$ Convergent

Hence, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$ is also convergent by the Comparison test.

4

(b) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2}$

$a_n = \frac{2n+1}{n^2}$, and $b_n = \frac{n}{n^2} = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2n+1}{n^2} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{2n+1}{n} = \lim_{n\to\infty} \frac{2+1/n}{1} = 2 > 0$$

$\sum_{n=1}^{\infty} \frac{1}{n} \rightsquigarrow$ Divergent Harmonic Series

$\therefore \sum_{n=1}^{\infty} \frac{2n+1}{n^2}$ is also divergent by the Limit Comparison Test.

v9.1

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2pt 1. Define precisely what is meant when we say that $\sum_{n=1}^{\infty} a_n$ converges to S.

An infinite series $\sum_{n=1}^{\infty} a_n$ converges to the sum S if its partial sums converge to S:

$$\lim_{N\to\infty} S_N = S \quad \text{where } S_N = a_1 + a_2 + \dots + a_N$$

4pts 2. Determine whether the following series converge or diverge:

(a) $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$

For $n \geq 2$, $n - \sqrt{n} \leq n$

$$\frac{1}{n-\sqrt{n}} \geq \frac{1}{n}$$

$\sum_{n=2}^{\infty} \frac{1}{n} \rightsquigarrow$ Divergent Harmonic Series

$\therefore \sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ is also divergent by the Comparison test.

4pts (b) $\sum_{n=2}^{\infty} \frac{2}{3^n \ln n}$

$a_n = \frac{2}{3^n \ln n}$, $b_n = \frac{1}{3^n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2}{3^n \ln n} \cdot \frac{3^n}{1} = \lim_{n\to\infty} \frac{2}{\ln n} = 0$$

$\sum_{n=2}^{\infty} \frac{1}{3^n} = \sum_{n=2}^{\infty} \left(\frac{1}{3}\right)^n \rightsquigarrow$ geometric series with $r = \frac{1}{3} < 1 \therefore$ convergent

Hence, $\sum_{n=2}^{\infty} \frac{2}{3^n \ln n}$ is also convergent by the Limit Comparison Test.

OR

We have $\frac{2}{3^n \ln n} < \frac{2}{3^n}$, for $n > 2$

Since $\sum_{n=2}^{\infty} \frac{2}{3^n}$ converges (geometric with $r = \frac{1}{3} < 1$), $\sum_{n=2}^{\infty} \frac{2}{3^n \ln n}$ also converges.

v9.2

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Math 1211: Quiz #9 **Winter 2016**

Name: SOLUTIONS	A#:	Section:

1. Define precisely what is meant when we say that $\sum_{n=1}^{\infty} a_n$ converges to S.

It means that $\lim_{N\to\infty}(a_1+a_2+\cdots+a_N) = S$

2. Determine whether the following series converge or diverge:

(a) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-\ln n}$

We have: $\frac{1}{\sqrt{n}-\ln n} > \frac{1}{\sqrt{n}}$

Since $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ diverges (p-series, $p=\frac{1}{2}<1$),

$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-\ln n}$ also $\boxed{\text{diverges}}$

(b) $\sum_{n=1}^{\infty} \frac{4^n}{3^n+5^n}$

We have: $\frac{4^n}{3^n+5^n} < \frac{4^n}{5^n} = \left(\frac{4}{5}\right)^n$

Since $\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$ converges (geometric with $r=\frac{4}{5}<1$),

$\sum_{n=1}^{\infty} \frac{4^n}{3^n+5^n}$ also $\boxed{\text{converges}}$