Math 1211: Worksheet #1 **Winter 2016**

Name: SOLUTIONS	A#:	Section:

1. $\int \frac{dx}{x \ln x}$ $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \ln|\ln x| + C$

2. $\int \sin 2x \cos^3 2x\,dx$ $u = \cos 2x$, $du = -\sin 2x \cdot 2\,dx$

$= -\frac{1}{2}\int u^3\,du$

$= -\frac{1}{2}\cdot\frac{1}{4}u^4 + C$

$= -\frac{1}{8}\cos^4 2x + C$

3. $\int_{-1}^{0} x^5\sqrt{x^3+1}$ $u = x^3+1 \Rightarrow du = 3x^2\,dx$

$\Rightarrow x^5 = x^3 \cdot x^2 = (u-1)\cdot\frac{1}{3}du$

$= \int_0^1 \frac{u-1}{3}\sqrt{u}\,du$

$= \frac{1}{3}\int_0^1 (u^{3/2} - u^{1/2})\,du$

$= \frac{1}{3}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Big|_0^1 = \frac{1}{3}\left(\left(\frac{2}{5}-\frac{2}{3}\right)-0\right) = \boxed{\frac{-4}{45}}$

4. $\int x \cot x^2\,dx$

Let $u = x^2$ so $du = 2x\,dx$: $\int x \cot x^2\,dx = \frac{1}{2}\int \cot u\,du$

$= \frac{1}{2}\int \frac{\cos u}{\sin u}\,du$

$= \frac{1}{2}\ln|\sin u| + C$

$= \frac{1}{2}\ln|\sin x^2| + C$

5. $\int \frac{dx}{\sqrt{9-4x^2}}$

$= \frac{1}{3}\int \frac{dx}{\sqrt{1-\frac{4}{9}x^2}}$ Let $u = \frac{2}{3}x$ so $du = \frac{2}{3}dx$

$= \frac{1}{3}\cdot\frac{3}{2}\int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{2}\sin^{-1} u + C$

$= \frac{1}{2}\sin^{-1}\left(\frac{2}{3}x\right) + C$

6. $\int \frac{x+4}{x^2+4}\,dx$

$= \int \frac{x}{x^2+4}\,dx + 4\int \frac{dx}{x^2+4}$

$= \frac{1}{2}\ln(x^2+4) + \int \frac{dx}{\frac{x^2}{4}+1}$ Let $u = \frac{x}{2} \Rightarrow du = \frac{1}{2}dx$

$= \frac{1}{2}\ln(x^2+4) + 2\int \frac{du}{u^2+1}$

$= \frac{1}{2}\ln(x^2+4) + 2\tan^{-1}(u) + C$

$= \frac{1}{2}\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + C$

7. $\int x\cos 3x\,dx$ [*Hint:* Integration by parts.]

$u = x$, $dv = \cos 3x\,dx$

$du = dx$ $v = \frac{1}{3}\sin 3x$

$\Rightarrow \int x\cos 3x\,dx = \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$

$= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C$

v1.1

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1. $\int \frac{dx}{x(\ln x)^2}$ $u = \ln x, \quad du = \frac{1}{x}dx$

$= \int \frac{du}{u^2}$

$= -\frac{1}{u} + C$

$= \boxed{-\frac{1}{\ln x} + C}$

2. $\int \sec^2 2x \tan^3 2x \, dx$ $u = \tan 2x, \quad du = \sec^2 2x \cdot 2\,dx$

$= \frac{1}{2}\int u^3\,du$

$= \frac{1}{2}\cdot\frac{1}{4}u^4 + C$

$= \boxed{\frac{1}{8}\tan^4 2x + C}$

3. $\int \frac{e^{2x}}{\sqrt{1+e^x}}\,dx$ $u = 1+e^x, \quad du = e^x dx$

$= \int \frac{e^x}{\sqrt{1+e^x}} e^x\,dx$

$= \int \frac{u-1}{\sqrt{u}}\,du = \int \left(u^{1/2} - u^{-1/2}\right)du$

$= \frac{2}{3}u^{3/2} - 2u^{1/2} + C = \boxed{\frac{2}{3}(1+e^x)^{3/2} - 2(1+e^x)^{1/2} + C}$

4. $\int x \cot x^2\,dx$

$= \frac{1}{2}\int \cot u\,du$ $\leftarrow u = x^2, \quad du = 2x\,dx$

$= \frac{1}{2}\int \frac{\cos u}{\sin u}\,du$

$= \frac{1}{2}\int \frac{dv}{v}$ $\leftarrow v = \sin u, \quad dv = \cos u\,du$

$= \frac{1}{2}\ln|v| + C = \boxed{\frac{1}{2}\ln|\sin x^2| + C}$

v1.3

+v1.2

5. $\cancel{\int \frac{dx}{4x^2+25}}$ $\int \frac{dx}{4x^2+25}$

$$= \frac{1}{25}\int \frac{dx}{\frac{4}{25}x^2+1}, \quad \text{let } u = \frac{2}{5}x, \; du = \frac{2}{5}dx$$

$$= \frac{1}{25}\cdot\frac{5}{2}\int \frac{du}{u^2+1}$$

$$= \frac{1}{10}\tan^{-1}(u) + C$$

$$= \boxed{\frac{1}{10}\tan^{-1}\left(\frac{2}{5}x\right) + C}$$

6. $\int \frac{1-x}{\sqrt{4-x^2}}\,dx$ [*Hint:* Split it into two integrals.]

$$= \int \frac{dx}{\sqrt{4-x^2}} - \int \frac{x}{\sqrt{4-x^2}}dx \qquad u = 4-x^2, \; du = -2x\,dx$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{1-\frac{x^2}{4}}} + \frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$$v = \frac{x}{2}, \; dv = \frac{1}{2}dx$$

$$= \int \frac{dv}{\sqrt{1-v^2}} + \frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$$= \sin^{-1}v + \sqrt{u} + C$$

$$= \boxed{\sin^{-1}\left(\frac{x}{2}\right) + \sqrt{4-x^2} + C}$$

7. $\int x\sin 3x\,dx$ [*Hint:* Integration by parts.]

$$u = x, \quad dv = \sin 3x\,dx$$

$$du = dx, \quad v = -\frac{1}{3}\cos 3x$$

$$\int x\sin 3x = -\frac{1}{3}x\cos 3x - \int\left(-\frac{1}{3}\cos 3x\right)dx$$

$$= \boxed{-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C}$$

v1.3

+v1.2