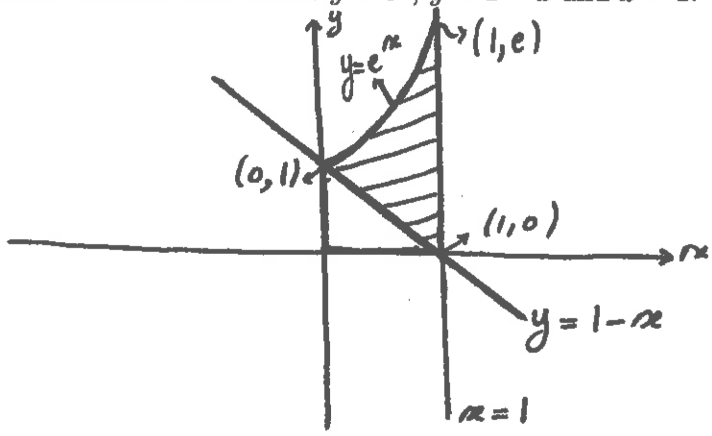
Math 1211: Worksheet #6 Winter 2016

Name: Ray	A#:	Section:

1. Let $\mathcal{R}$ be the region bounded between the curves $y = e^x$, $y = 1 - x$ and $x = 1$.

   (a) Sketch $\mathcal{R}$.



   (b) Give expressions, in terms of definite integrals, for the following.
   Do not evaluate your integrals!

Shell Method

   i. The volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis.

$$V = 2\pi \int_0^1 x \left[e^x - (1-x)\right] dx$$

Radius = x

height = $e^x - (1-x)$

Shell Method

   ii. The volume of the solid obtained by revolving $\mathcal{R}$ about the line $y = 2$.

$$V = 2\pi \int_0^1 (2-x)\left[e^x - (1-x)\right] dx$$

Radius = $2-x$

height = $e^x - (1-x)$

Washer Method

$R_i = 1-x$

$R_o = e^x$

   iii. The volume of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

$$V = \pi \int_0^1 (e^x)^2 - (1-x)^2 \, dx$$

   iv. The surface area of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

$$SA = 2\pi \int_0^1 e^x \sqrt{1+(e^x)^2}\, dx + 2\pi \int_0^1 (1-x)\sqrt{2}\, dx + \pi e^2$$

2. Find the length of the curve $y = \ln(\cos x)$ between $x = 0$ and $x = \pi/4$.

$$y = \ln(\cos x)$$

$$y' = \frac{1}{\cos x}(-\sin x) = -\tan x$$

$$S = \int_0^{\pi/4} \sqrt{1 + (-\tan x)^2}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/4} \sec x \, dx$$

$$= \ln|\sec x + \tan x| \Big]_0^{\pi/4} = \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right| - \ln|\sec 0 + \tan 0|$$

$$= \ln(\sqrt{2} + 1)$$

3. Consider the parametric curve C given by $(x, y) = (e^t - 1, e^{2t} + 1)$ for $0 \le t < \infty$.

(a) Find the Cartesian equation (i.e. $y = f(x)$) form of C.

$$x = e^t - 1, \quad y = e^{2t} + 1$$

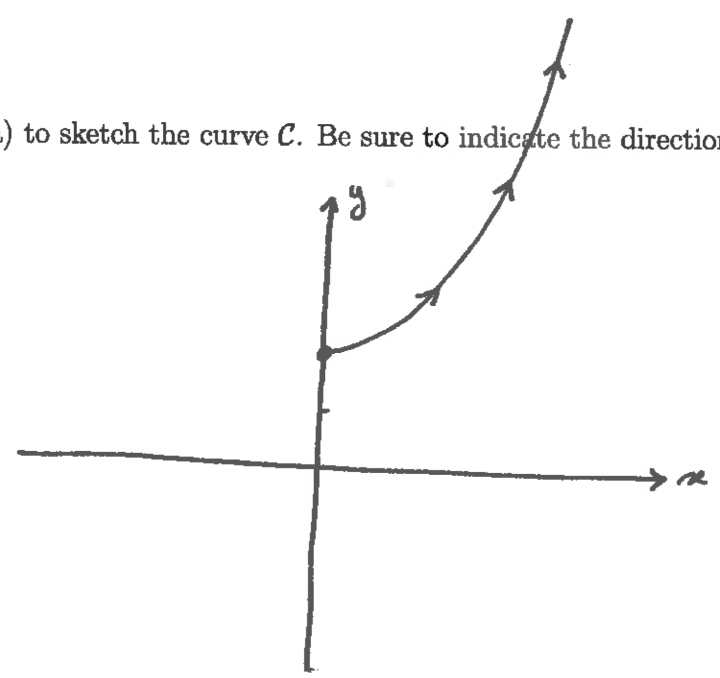
$$x = e^t - 1 \Rightarrow e^t = x + 1$$

$$y = e^{2t} + 1 \Rightarrow y = (e^t)^2 + 1$$

$$= (x+1)^2 + 1$$

$$\therefore\ y = x^2 + 2x + 2, \quad x \geqslant 0$$

(b) Use (a) to sketch the curve C. Be sure to indicate the direction of travel.



Math 1211: Worksheet #6 Winter 2016

Name: Key	A#:	Section:

1. Let $\mathcal{R}$ be the region bounded between the curves $y = 4 - x^2$, $y = 3x$ and $x = 0$.

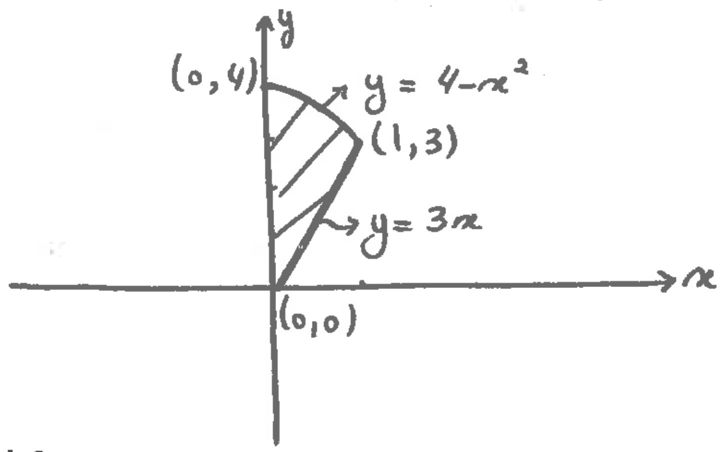
   (a) Sketch $\mathcal{R}$.

$3x = 4 - x^2$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4$ or $x = 1$



   (b) Give expressions, in terms of definite integrals, for the following.
   Do not evaluate your integrals!

   i. The volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis.

Shell Method:

Radius $= x$

height $= (4 - x^2) - 3x$

$$V = 2\pi \int_0^1 x\left[(4 - x^2) - 3x\right] dx$$

   ii. The volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = 3$.

Shell Method:

Radius $= 3 - x$

height $= (4 - x^2) - 3x$

$$V = 2\pi \int_0^1 (3 - x)\left[(4 - x^2) - 3x\right] dx$$

   iii. The volume of the solid obtained by revolving $\mathcal{R}$ about the line $y = -2$.

Washer Method:

$R_i = 2 + 3x$

$R_o = 2 + (4 - x^2)$

$$V = \pi \int_0^1 \left(\left[2 + (4 - x^2)\right]^2 - (2 + 3x)^2\right) dx$$

   iv. The surface area of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

$$SA = 2\pi \int_0^1 (4 - x^2)\sqrt{1 + 4x^2}\, dx + 2\pi \int_0^1 3x\sqrt{1 + 3^2}\, dx + 16\pi$$

2. Find the length of the curve $y = \ln(\sin x)$ between $x = \pi/4$ and $x = \pi/2$.

$y = \ln(\sin x)$

$y' = \dfrac{\cos x}{\sin x} = \cot x$

$S = \displaystyle\int_{\pi/4}^{\pi/2} \sqrt{1+\cot^2 x}\, dx$

$= \displaystyle\int_{\pi/4}^{\pi/2} \sqrt{\csc^2 x}\, dx$

$= \displaystyle\int_{\pi/4}^{\pi/2} \csc x \, dx$

$= \ln\left|\csc x - \cot x\right| \Big]_{\pi/4}^{\pi/2} = \ln\left|\csc\frac{\pi}{2} - \cot\frac{\pi}{2}\right| - \ln\left|\csc\frac{\pi}{4} - \cot\frac{\pi}{4}\right|$

$= \ln|1-0| - \ln\left|\sqrt{2}/2 - 1\right|$

$= -\ln(\sqrt{2}-1)$

3. Consider the parametric curve C given by $(x, y) = (\cos 2t, \cos t)$ for $0 \leq t \leq 2\pi$.

(a) Find the Cartesian equation (i.e. $y = f(x)$) form of C.
Hint: Use a trigonometric identity.

$x = \cos 2t$, $y = \cos t$

$x = \cos 2t = 2\cos^2 t - 1$

$\Rightarrow 2\cos^2 t = 1 + x$

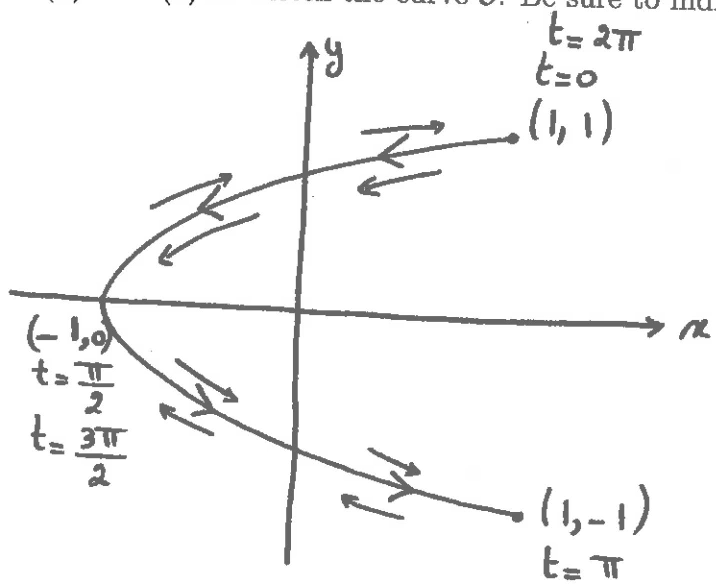
$\Rightarrow \cos^2 t = \dfrac{1+x}{2}$

Now $y = \cos t$

$\Rightarrow y^2 = \cos^2 t$

$\therefore y^2 = \dfrac{1+x}{2}$ or $2y^2 = 1 + x$

(b) Use (a) to sketch the curve C. Be sure to indicate the direction of travel.



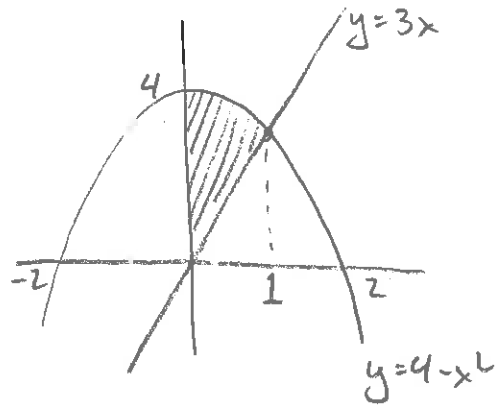
v6.2

Math 1211: Worksheet #6 **Winter 2016**

Name: SOLUTIONS	A#:	Section:

1. Let $\mathcal{R}$ be the region bounded between the curves $y = 4 - x^2$, $y = 3x$ ~~and $x \geq 0$~~. to the right of the y-axis

   (a) Sketch $\mathcal{R}$.



Note: $4 - x^2 = 3x$
$\iff x^2 + 3x - 4 = 0$
$\iff (x+4)(x-1) = 0$
$\iff x = -4$ OR $x = 1$

   (b) Give expressions, in terms of definite integrals, for the following.
   Do not evaluate your integrals!

      i. The volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis.

$$2\pi \int_0^1 x(4 - x^2 - 3x)\,dx$$

      ii. The volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = 3$.

$$2\pi \int_0^1 (3 - x)(4 - x^2 - 3x)\,dx$$

      iii. The volume of the solid obtained by revolving $\mathcal{R}$ about the line $y = -2$.

$$\pi \int_0^1 \left((4 - x^2 + 2)^2 - (3x + 2)^2 \right) dx$$

      iv. The surface area of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

$$2\pi \int_0^1 (4 - x^2)\sqrt{1 + (-2x)^2}\,dx + 2\pi \int_0^1 3x\sqrt{1 + 3^2}\,dx + \pi \cdot 4^2$$

$$\left(= 2\pi \int_0^1 (4 - x^2)\sqrt{1 + 4x^2}\,dx + 3\sqrt{10}\,\pi + 16\pi \right)$$

v6.3

2. Find the length of the curve $y = \ln(\sin x)$ between $x = \pi/4$ and $x = \pi/2$.

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$$

$$\text{So Length} = \int_{\pi/4}^{\pi/2} \sqrt{1+\cot^2 x}\, dx$$

$$= \int_{\pi/4}^{\pi/2} \sqrt{\csc^2 x}\, dx$$

$$= \int_{\pi/4}^{\pi/2} \csc x\, dx$$

$$= -\ln|\csc x + \cot x| \Big]_{\pi/4}^{\pi/2} = -\ln 1 + \ln(\sqrt{2}+1)$$

$$= \boxed{\ln(\sqrt{2}+1)}$$

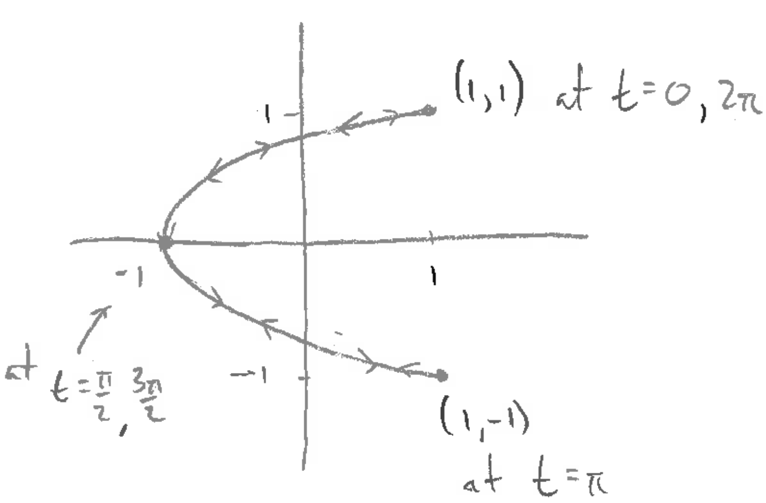
3. Consider the parametric curve C given by $(x, y) = (\cos 2t, \cos t)$ for $0 \le t \le 2\pi$.

(a) Find the Cartesian equation (i.e. $y = f(x)$) form of C.
Hint: Use a trigonometric identity.

$$x = \cos 2t$$
$$= 2\cos^2 t - 1$$
$$= 2y^2 - 1$$

So $\boxed{x = 2y^2 - 1}$ But this comes with the restriction that $-1 \le y \le 1$, since $y = \cos t$.

(b) Use (a) to sketch the curve C. Be sure to indicate the direction of travel.



DIRECTION OF TRAVEL

v6.3