Name: SOLUTIONS	A#:	Section:

1. Find the 4-th order Maclaurin polynomial of $f(x) = \sin x - \cos x$.

$$f(x) = \sin x - \cos x \Rightarrow f(0) = -1$$
$$f'(x) = \cos x + \sin x \Rightarrow f'(0) = 1$$
$$f''(x) = -\sin x + \cos x \Rightarrow f''(0) = 1$$
$$f'''(x) = -\cos x - \sin x \Rightarrow f'''(0) = -1$$
$$f^{(4)}(x) = \sin x - \cos x \Rightarrow f^{(4)}(0) = -1$$

So $$M_4(x) = -1 + 1\cdot x + \frac{1}{2!}x^2 + \frac{(-1)}{3!}x^3 + \frac{(-1)}{4!}x^4$$
$$= \boxed{-1 + x + \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{24}}$$

2. Find the n-th order Taylor polynomial of $f(x) = \dfrac{1}{(1-2x)^2}$ centred at -1.

$$f(x) = (1-2x)^{-2}$$
$$f'(x) = (-2)(1-2x)^{-3}(-2)$$
$$f''(x) = (-2)(-3)(1-2x)^{-4}(-2)^2$$
$$f'''(x) = (-2)(-3)(-4)(1-2x)^{-5}(-2)^3$$
$$\vdots$$
$$f^{(k)}(x) = (-1)^k\cdot(k+1)!\,(1-2x)^{-(k+2)}(-2)^k$$
$$= 2^k(k+1)!\,(1-2x)^{-(k+2)}$$

So $$f^{(k)}(-1) = 2^k(k+1)!\cdot 3^{-(k+2)} \Rightarrow \frac{f^{(k)}(-1)}{k!} = (k+1)\frac{2^k}{3^{k+2}}$$

and $$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(-1)}{k!}(x+1)^k$$
$$= \sum_{k=0}^{n} (k+1)\frac{2^k}{3^{k+2}}(x+1)^k$$
$$= \frac{1}{9} + \frac{4}{27}(x+1) + \frac{4}{27}(x+1)^2 + \cdots + (n+1)\frac{2^n}{3^{n+2}}(x+1)^n$$

3. Recall that the n-th order Maclaurin polynomial of e^x is

$$M_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}.$$

Use this to find the Maclaurin polynomials of the following functions to the given order:

(a) $f(x) = 3e^x + 1$ [order 4]

$$3\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right) + 1 = 4 + 3x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{8}$$

(b) $g(x) = e^x + e^{-x}$ [order 4]

$$\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right) + \left(1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \frac{(-x)^4}{4!}\right)$$

$$= 2 + x^2 + \frac{x^4}{12}$$

(c) $h(x) = e^{-x^2}$ [order 8]

$$1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!}$$

$$= 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!}$$

(d) $k(x) = x^3 e^{x^2}$ [order 8]

$$x^3\left(1 + x^2 + \frac{(x^2)^2}{2!}\right)$$

$$= x^3 + x^5 + \frac{x^7}{2!}$$