Nova Scotia Math League

2013–2014

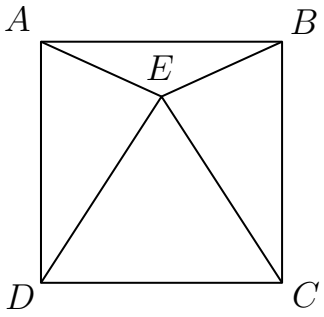
Game Three

Contest Paper

Team Questions

1. What is the remainder when $\overbrace{1111111111\cdots111}^{\text{100 ones}}$ is divided by 4?

2. Point E lies inside square $ABCD$ such that $\triangle CDE$ is equilateral. Find the measure of $\angle AEB$, in degrees.



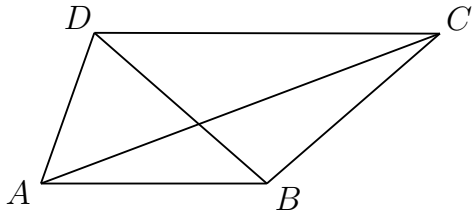
3. Suppose
$$x^2 - 2014x = y^2 - 2014y$$
for two different numbers x and y. Find the sum $x + y$.

4. Find the angle between the minute hand and hour hand of a clock at 12:12pm.

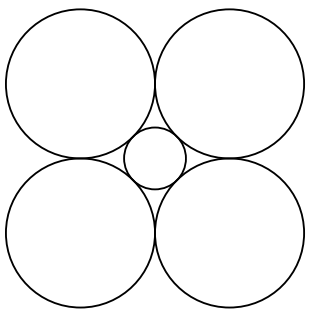
5. How many palindromes can be formed by rearranging the letters of the word MISSISSIPPI?

   Note: A palindrome is a word that reads the same both forward and backward, such as RACECAR.

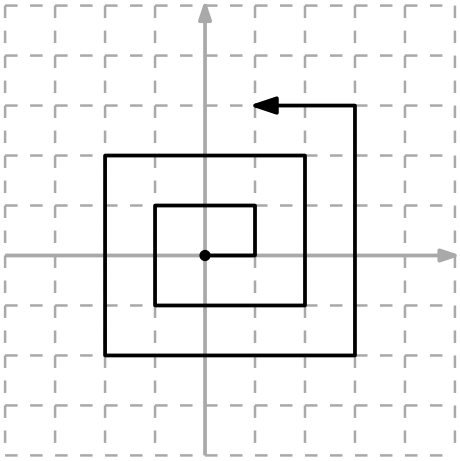
6. Sides AB and CD of quadrilateral $ABCD$ are parallel, and $|BA| = |BD| = |BC|$. If $\angle CAB = 20^\circ$, find $\angle DAC$.



7. Four identical circles are tangent to a smaller circle and also to each other, as shown below. Find the ratio of the diameter of a larger circle to the diameter of the smaller circle.

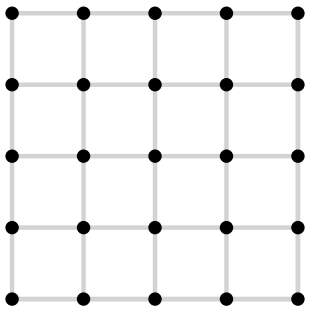


8. An ant finds itself at the origin $(0, 0)$ of a unit grid. The ant begins walking in a "square spiral" as illustrated below. What is the total distance travelled by the ant when it reaches the point $(10, 0)$?



9. Three six-sided dice are rolled. What is the probability that their sum is divisible by 6?

10. A 5×5 square grid contains 25 lattice points, as shown in the diagram below. Find the number of pairs $\{P, Q\}$ of distinct lattice points such that the midpoint of segment PQ is also a lattice point.



Pairs Relay

P-A. Exactly 12 different numbers can be obtained by rearranging the digits of 5667. Let `A` be the number of these that are divisible by 15.

Pass on `A`

P-B. You will receive `A`.

Increasing the diameter of a circle by `A` units quadruples its area.

Let `B` be the original diameter of the circle.

Pass on `B`

P-C. You will receive `B`.

The average of a list of `B` numbers is 6, and the average of a list of 6 numbers is `B`. Let `C` be the average of all the numbers when the two lists are combined.

Pass on `C`

P-D. You will receive `C`.

Points P, Q, R, and S lie on a line (in that order) such that

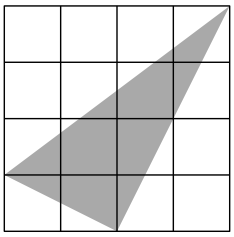
$$|PQ| : |QR| = |QR| : |RS| = 6 : \texttt{C}.$$

Let $D = \dfrac{|PR|}{|RS|}$.

Done!

Individual Relay

I-A. Each square in the grid below has an area of 1 square unit. Let A be the area of the shaded triangle, measured in square units.



Pass on A

I-B. You will receive A.

A bottle of chocolate syrup will produce 2 litres of chocolate milk when mixed in a ratio of 1 part syrup to 3 parts milk. If, instead, it is mixed in a ratio of 1 part syrup to A parts milk, then it will produce B litres of chocolate milk.

Pass on B

I-C. You will receive B.

The function f is defined by $f(x) = \dfrac{x+1}{x-1}$ for all $x \neq 1$.

Let C $= f(f(f(f(f(\mathrm{B})))))$.

Pass on C

I-D. You will receive C.

The ratio $x : y$ is $3 : \mathrm{C}$. Let D $= \dfrac{x-y}{x+y}$.

Done!