Response to the "Foundation for the Atlantic Canada Mathematics Curriculum, Validation Draft" (October 1995).

A report prepared for the Mathematics subcommittee of the
Atlantic Provinces Inter-University Council for the Sciences (APICS)

by

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The stated Essential Graduation Learnings and the four Unifying Concepts for Curriculum Outcomes are laudable goals. This response addresses the four Content Strands and the first Unifying Strand (Problem Solving).

The Appendix addresses more specific issues relating to the proposed grades 10-12 courses 1-4 (or 10-40), circulated to some members of APICS at provincial-level information sessions.

The students of primary concern to APICS are, naturally, those who will go on to study university-level mathematics, science and engineering. The majority of students do not pursue such a course. This document addresses the needs of both the future scientists and engineers and the needs of mainstream students.

The advice herein can be briefly summarized as follows.

- The present document does not adequately emphasize the importance of fractions and algebra.
- The topics covered must be basic to mathematics. Superficial exposure to a difficult topic is extremely difficult for teachers to accomplish: either students will be confused, or they will have a false sense of their own abilities. Teachers must be comfortable with topics in the curriculum. In particular, teachers should understand the topic to a greater depth than that expected of the students.
- The curriculum document should clearly indicate which topics are essential to progress in mathematics.

Unifying Strand 1: Number Concepts/Number and Relationship Operations.

Numbers are the building blocks of K-12 mathematics, and the logical first topic for a curriculum document.

(a) The list of topics under "Number Concepts ..." on pages 14ff create the impression that whole numbers and fractions are different number systems.

Surely, a primary emphasis of the mathematics program should be exploration of the real number system, with particular attention to the four operations (+,-,x,/). In the early grades, emphasis is on the whole numbers. At some point, students should be encouraged to notice that the number line represents numbers other than whole numbers, so that fractions may be introduced. ... Rules for arithmetic with negatives, fractions and irrationals should be presented as extensions of what has come before, not as new systems. Finally, when a student has mastered the rules for arithmetic on the real line, algebra can be introduced: an abstract treatment of the same old rules for arithmetic. Algebra is the language of mathematical Problem Solving, the primary Unifying Concept of this curriculum document.

The academic students will eventually work out that whole numbers and fractions are special sub-systems of the real number system. It is the nonacademic students who are more likely to be confused by a lack of continuity in the presentation of numbers.

(b) <u>Nowhere</u> in this document is there a statement which says: by this point, students can perform all four operations on fractions, including rather complicated fractions, and complex combinations of operations.

There are references to decimals, and to modelling and estimation, but we can find no reference to dexterity of arithmetic with fractions. Without this basic skill, students cannot grasp algebra, nor some of the basic functions applicable to the modelling of problems. Any educator who believes that decimal-based calculators have made rational arithmetic superfluous has misunderstood the value of algebra.

We suggest that a clear time-frame be stated for mastery of arithmetic with fractions. A logical target date would be one on which students "graduate" to a new school. For example, if most students in Atlantic Canada change schools after the sixth grade, mastery of fractions could be a requirement for completion of grade 6. While we are aware that some educators would set the seventh grade as such a target, we are obliged to point out that academic students need some exposure to fractions in grades 4-6. With particular reference to fractions, we also suggest that Ministries of Education study the mathematical goals of other industrialized countries.

(c) It is not clear when, if ever, students gain experience with manipulation of exponents and radicals. Does item 26 (page 25) imply such skill development through practice? Surely the topic should be treated in greater depth in grades 10-12, and revisited in Strand 2?

Unifying Strand 2: Patterns and Relations.

Functions lie at the heart of mathematics. Both academic and nonacademic students need exposure to functions.

(a) We caution (as in Strand 4, below) that students' limited experiences of the physical world may make pattern recognition difficult. It would certainly be easy to confuse them. For example, students at all levels will have difficulty understanding that "a cube has six faces" is a fact (in particular, there is no "doubt" associated with this statement), whereas "weight increases with height" is a conclusion that can be drawn from empirical evidence (which is always associated with "noise" or doubt). In particular, there will be a child in most classes who seems to break the weight:height rule. But you will never find a cube that does not have six faces.

Some material on graphing and relations is covered in other science courses, where it is easier to motivate. There is no need to import more material into a crammed curriculum.

(b) There should be a specific list of functions, and a time frame for their introduction. For example, straight lines should be introduced before the end of grade 9. When should quadratics be introduced? Polynomials? Will log and exponential functions be included? These functions are important in many applications of mathematics.

(c) References to sequences, series, limits are too vague. Examples from business are appropriate for all students. Nonacademic students find these topics very difficult.

Unifying Strand 3: Shape and Space.

Constant reference to diagrams, and to the world around, are essential "connections" in any mathematics program.

(a) Some of the K-3 goals are ambitious.

(b) All graduating students should have some exposure to coordinate geometry and trigonometry. We see no problems here, though the academic students need more exposure than do the nonacademic students.

(c) Do items 85, 86 (page 30) refer to "Theorem, to prove, proof" geometry? The better academic students would benefit from such rigor. Nonacademic students would be frustrated.

Unifying Strand 4: Data Management and Probability.

There is no denying that the media bombards us with the results of surveys, and other data-based inquiries. This blatant evidence of mathematics-around-us has, perhaps, influenced the writers of school curricula, so that there is an increasing emphasis on statistics.

(a) The single most important statistical concept which should be grasped by graduating high school students (both academic and nonacademic) is that any report of a data-based decision should include both a description of the manner in which data were collected, and an estimate of the doubt associated with the final decision.

(b) How can we expect third graders to be able to "formulate and solve problems that involve collecting and analyzing data"? At the very least, we suggest that the material on page 31 be moved to the right. Thus, the column labeled "By the end of Grade 3" should move to "By the end of Grade 6", and the final column should disappear.

Note, in particular, that the Normal distribution would then be dropped. Although academic students could handle a simple introduction to the Binomial distribution, the Normal can only be tackled (at that level) as a miracle of nature. Furthermore, most teachers would have no feeling for situations where the Normal distribution does not apply.

(c) The single biggest error which professional statisticians see non-professionals make is the application of an inappropriate technique (leading to an underestimation of error rate). A little statistics is a dangerous thing. We must be sure that our teachers can point to the limited applicability of any techniques introduced at this level.

(d) Probability is an extremely difficult topic.

(e) There is far too much emphasis on simulation.

Unifying Concept 1: Problem Solving.

There is no denying the importance of problem solving. This is, certainly, the goal of mathematics education.

(a) In first-semester university calculus, there are two classic types of applied problem (min/max and related rates). University teachers know that students find these problems difficult. There are two difficulties: expression of the problem in mathematical terms, and manipulation of symbols to obtain a solution. That is to say, the difficulties arise from inexperience with the language of mathematics: algebra. Students cannot grasp multiplication until they have addition facts "at their fingertips". They cannot do arithmetic with fractions until they have multiplication facts "at their fingertips". Similarly, they cannot grasp algebra until they have mastered fractions. And they will be swamped by the technicalities of problem-solving if they are not dexterous with the rules of algebra. Many of our students spend hours practicing other disciplines (hockey, basketball, piano, singing, mountain-climbing, for example). We maintain that students will not master the art of problem-solving if they have not mastered the prerequisite skills.

(b) With reference to the discussion on lines 10-17 of page 10, we make two points.

1. Traditional word problems are essential training for more difficult problems.

2. We trust that teachers will bear in mind their students' limited experience of the world. For example, we must not lead school children to believe that, with their limited experience and knowledge, they can actually solve the world's environmental problems. At best, they can grasp some of the many factors involved.

Appendix: courses 1-4 (10-40)

The idea of a function tool-kit is very good, especially if it involves considerable hand and calculator work with the whole range of special functions. The idea of doing more concrete geometry is also good.

The gravest concerns are these.

1. No room in the courses for familiarization with basic skills.

2. No clear statements on relative importance of such a large volume of material.

   Surely many of these topics are optional extras, depending on the mathematical background of the teacher.

3. Overemphasis of discrete mathematics and difficult statistical concepts.

   Many of the topics listed in that category are difficult even for third year university students, even for their professors. In fact, many mathematics professors admit that they cannot do a decent job in statistics. The methodology and standard ideas are just *too* different.

   So how can we ask high school teachers to cope with the more sophisticated statistical techniques listed in the Framework? Or are they to be treated in only a superficial way? If so, then stop!

4. A half-hearted introduction to calculus.

   Are these four courses not intended as standard fare for almost all high school students? Can we expect all of them to cope with calculus? No. It is better to build a solid foundation (algebra, trigonometry, and functions) than to reach beyond the mainstream students' grasps.

5. The need for (at the very least) another high school mathematics course for those students heading towards university level science and engineering.

The four course outlines cover far more material than does the curriculum document. The mood in education circles seems to be that more students should be passing mathematics, but these outlines include difficult material, beyond the grasp of the average (nonacademic) student. Tentative or not, these course descriptions could set some dangerous directions.

Some explicit suggestions:

Course 1:

(a) It makes more sense to put least squares in Course 4, with curve fitting.

(b) Remove statistical packages and experimental design. It is just too difficult to present problems which students *can* tackle, without creating a false sense of security: most statistical problems are too difficult for them to handle.

(c) Matrices are of little use at this stage. Perhaps the authors were thinking of spreadsheets as matrices. But matrices should be introduced at a time when matrix arithmetic is of use, for example in the unit on vectors in course 3.

(d) Similarly, under Relations and Functions, remove linear programming and also transformation matrices. Moreover, the transformation of quadrics is misplaced: surely the next course is a better place.

Course 2:

(a) A Caution to Teachers: probability is *very* difficult to teach. This is an extremely dangerous area, although potentially worthwhile and interesting.

(b) Some of the Discrete Math topics are, however, of little benefit. There is no point to introducing new concepts or notation when the mainstream student can understand only superficial applications.

There is no use in doing graphs (here meaning connected dot diagrams), except perhaps for "trees", which are useful in probability and elsewhere. Consequently, remove the matrix description of graphs – there really is no need for them or for Euler trails.

Yes, many things are nice or pretty, but none is crucial and few have significant applications, even at university level. There is not enough time to waste on graphs.

(c) Matrices and Markov chains do not belong here. If anywhere, they should follow conditional probability in Course 3.

(d) The normal, central limit theorem, chi square, etc. are all of major importance in the world around us. But these topics are too technical for anything but superficial (and possibly misleading) consideration in high school.

If the goal is to develop some sensible appreciation of statistical arguments, then throw away all this complicated machinery. Introduce the Binomial and the sign test, but preferably when the students are (mathematically speaking) a little older.

Course 3:

(a) The item on convergence is dubious, unless the traditional topic of geometric series is meant. Few second year university students can cope with much of this.

(b) The topic of log regression and non-linear data again is inappropriate. But if it does survive, it must come in the later course.

(c) Relations and Functions: square root could better go in course 2 – why not swap with the exponentials in course 2? This would bring all the material on exponential and logarithmic functions together into a more coherent package.

<u>Course 4:</u>

(a) The folly of stuffing in bits and pieces of calculus has been addressed above.

(b) The traveling salesman problem makes an interesting diversion, but does not deserve special or extended treatment. Of course, a good teacher brings in many highlights and novel applications– but these do not belong in a framework document.

(c) Optional groups and fields: see remarks on traveling salesman. Do not mention groups and fields in a foundation document.

(d) Euler's formula goes better with the 3-dimensional geometry in Course 1.