THE ATLANTIC HIGH SCHOOL MATHEMATICS CURRICULUM: IMPLICATIONS FOR UNIVERSITIES

A Report of the Mathematics/Statistics Committee of The Atlantic Provinces Council on the Sciences

Endorsed by APICS Council, April 13, 2002

In today's world, economic access and full citizenship depend crucially on math and science literacy. I believe that the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of registered black voters in Mississippi was in 1961 . . . [M]ath literacy—and algebra in particular—is the key to the future of disenfranchised communities.

Radical Equations: Math Literacy and Civil Rights, *Robert P. Moses and Charles E. Cobb Jr., Beacon Press, 2001*

1 Preamble

In September 2001, over 3500 students registered in calculus courses at Atlantic Canada universities while another 600 were prevented from doing so because of inadequate basic mathematical skills. Failure rates in first year calculus courses have been high for years (roughly 25%, on average). This is discouraging and highly nonproductive for students and professors alike.

During the Fall 2001 semester, Memorial University tracked the progress of 55 first year calculus students all of whom had followed a new mathematics curriculum which is being introduced into the high schools of the four Atlantic Provinces. Nearly sixty percent of these students either failed or withdrew during the semester. Whether compared with students from the same schools or students across Newfoundland, this figure is roughly double the attrition rate of students from the former curriculum. This could be a one-time glitch, but it does raise a concern about the preparation of students entering programmes in business, engineering, science, pharmacy and other disciplines for which calculus is a prerequisite.

In late December 2001, Dr. Roger Gordon, Dean of Science at the University of Prince Edward Island and Chair of the Atlantic Provinces Council on the Sciences (APICS), wrote Dr. Edgar Goodaire, Chair of the APICS Mathematics/Statistics Committee and a former head of the Department of Mathematics and Statistics at Memorial University, expressing concern over the revisions that have been taking place in the high school mathematics curriculum in Atlantic Canada. Specifically, Dr. Gordon enquired about admission requirements to university science programmes and the content of first year calculus courses. In each case, the question was "Will changes be needed?"

Early in 2002, Dr. Gordon returned to the Committee formally requesting a report which would address additional questions.

- Is the new curriculum common across the region?
- When will the new curriculum be implemented?
- Have universities other than Memorial studied the effects of the new curriculum on university performance?
- What can APICS do to minimize the apparently negative effects of the new curriculum on first year science students?

Here are the short answers to most of Dr. Gordon's questions. We defer suggestions about what APICS (and others) can do to final sections of this report. In this report, readers should note that references to "calculus" courses are to those required by programmes in a Faculty of Science or Engineering and, by "admission requirements," we mean "admission to calculus courses."

2 Some Answers

2.1 Is the new curriculum common across the region?

Yes it is, although course numbers vary from province to province, as do the distribution of the material and the resources being used. New Brunswick has three courses (two in Grade 11, one in Grade 12) which correspond to two courses in Nova Scotia and Newfoundland. New Brunswick, Nova Scotia and Newfoundland are making use of textbooks produced by a team of writers under the direction of the Atlantic Provinces Education Foundation (APEF). While Prince Edward Island is also adopting the common curriculum, it made a decision in June 1998 to choose its own resources and to develop its own process for implementation, separately from the other provinces. (For more detail, see Appendix C.)

2.2 When will the new curriculum be implemented?

The implementation of the curriculum has varied from province to province. There were pilot projects in selected schools in different areas beginning as early as 1998. The "Grade 10" years which appear in Table 1 are those by which most (perhaps all) schools in a province were using the new curriculum. "First university cohort" means first **full** cohort.

2.3 Pilot studies

To date, only Memorial has tracked the university performance of students who have experienced the full three years of the new curriculum and made comparisons with success rates of students from the former curriculum.

Table 1: Implementation

	Grade 10	*First university cohort expected*
Newfoundland	2000-2001	Fall 2003
New Brunswick	2001-2002	Fall 2004
Nova Scotia	1999-2000	Fall 2002
Prince Edward Island	2000-2001	Fall 2003

3 The Curriculum

3.1 Pro

Proponents of the courses based on the new curriculum feel strongly that the new curriculum and the materials which support it are based on sound pedagogical principles and that the new approach should be given a chance. They like its focus on discovery-based learning techniques. Much of the criticism of the new curriculum focuses on the textbooks that were written to accompany it. While these texts indeed emphasize the discovery approach, they are not the only resource available to students. More traditional material from the former curriculum will still be found in departmental offices and from educational suppliers.

Advocates for the new approach stress that the basic algebra, trigonometry, geometry, and theory of functions, topics in which universities want students to become proficient, **are** covered in the new curriculum and in a way that will improve students' understanding of them. We refer the reader to Appendix B for more details of the history, content and implementation of the new curriculum, presented from a sympathetic viewpoint.

3.2 Con

University professors have been involved in providing feedback to the development of the new mathematics curriculum and have commented at length on drafts of text books. Some professors, however, feel they have been ignored and that their considerable efforts were made largely in vain. Concerns about lack of precision and care in the preparation of the materials, as well as errors in both theory and illustrations, were simply not acknowledged, let alone addressed.

At least twice in recent years, in October 1995 and again in May 2001, the APICS Mathematics/Statistics Committee has gone on record with its views on the proposed curriculum. These statements, prepared by Drs. Barry Monson and Maureen Tingley of the University of New Brunswick (Fredericton) and (in 1995) Dr. Mark Taylor of Acadia University, are available at `www.math.mun.ca/~apics/`.

The themes of these two documents are rather consistent.

- There is not enough emphasis on the mastery of basic skills from long division to addition of fractions and manipulation of exponents.
- A little statistics is a dangerous thing. Any discussion of this topic in the high schools should emphasize the importance of attaching to numbers an estimate of their accuracy and a description of the means by which data were collected.
- Computers and calculators should not be used as substitutes for the mastery of basic algebra skills.
- Calculus should be taught only on a solid foundation of algebra, trigonometry and functions.

Opponents of the new curriculum still claim that it places not nearly enough emphasis on the mastery of basic skills. (Astonishingly, a recent bulletin from the Department of Education in New Brunswick[1] appears to boast that "algebraic manipulations" are now left to optional courses!) Every university mathematics professor in the world will say that the ability to do arithmetic with fractions, without giving the matter much thought, and the instinct not to simplify rational expressions which cannot be simplified, are absolutely essential to success in calculus and later courses.

The new curriculum encourages (even more than the old) the use of calculators and manipulatives (physical objects intended to help visualise and model abstract ideas). Judicious use of both may help some students, but they can also hinder the ability to perform certain mathematical tasks automatically. The public perception that mathematics cannot be done without a calculator is a sign that mathematicians and mathematics educators have done a poor job explaining the difference between mathematics and arithmetic.

The content of Grade 10 is shallow and includes topics which are new to most teachers. Students will get through the year seeing few radicals, no reinforcement of arithmetic with fractions and very little manipulation of symbols. The year is ripe for the spread of math anxiety within the teaching ranks. Good students will be bored to death. They may also acquire bad habits and be unable to distinguish those topics which are useful and important (for calculus, for example) from those which are less so.

There is an over-emphasis on topics that used to be reserved for universities, at the expense of what have always been deemed core school topics absolutely essential for success in higher mathematics. From the perspective of many, the curriculum attaches far too little importance to good old-fashioned practice.

Over the years, curricula have expanded in breadth at the expense of depth. Some topics are treated very superficially. (Statistics is always raised as an example by professional mathematicians and statisticians.) Since some topics are hard to teach at university, we are concerned that school teachers may not have the background necessary to teach them with confidence in high school.

[1] February 22, 2002

4 What can be done?

In response to Dr. Gordon's query about what APICS can do to assist students entering faculties of science, we stress at the outset the importance of common standards and policies throughout Atlantic Canada. A unifying theme of the recommendations that follow is that universities and mathematics faculty in Atlantic Canada must stick together (with the staunch support of senior administrators).

The admission requirements for calculus which universities intend to impose in response to the new curriculum are shown in Table 2. Most Canadian universities require precalculus for their standard first semester science or engineering calculus course. Though we are fully aware that Memorial and UPEI do not currently require precalculus, we feel this is important.

Table 2: Requirements for Admission to Calculus under the New Curriculum

University	*Requirements*
Acadia	Math 10, Advanced Math 11 and 12, precalculus and adequate performance on a placement test
Dalhousie	Math 10, Advanced Math 11 and 12, precalculus
Memorial	(Advanced) Math 2205 and 3205 and satisfactory performance on a placement test
Mount Allison	Math 10, Math 111 or 112 (two semesters), Math 121 or 122, Advanced Math 120
UNB	Math 10, Math 111 or 112 (two semesters), Math 121 or 122, Advanced Math 120 and satisfactory performance on a placement test
UPEI	Math 421A, 521A, 621A and, for students who will take more than one semester of calculus, satisfactory performance on an assessment test
St. Francis Xavier	Math 10, advanced math 11 and 12, precalculus
St. Mary's	Math 10, Advanced Math 11 and 12, precalculus and adequate performance on a placement test

Recommendation 1. *For admission to a calculus course in a Faculty of Science, every university in Atlantic Canada should require completion of all "advanced" courses (including the precalculus course) offered under the new curriculum (see Appendix A for course names and numbers in each province).*

This recommendation implies that students wishing to study calculus for the purposes of a science degree at university complete courses in their respective provinces as shown in Table 3.

Table 3: High School Courses Recommended for University Calculus

Province	*Recommended Courses*
New Brunswick	Math 10, Math 111 or 112 (2 courses), Math 121 or 122, Math 120
Newfoundland	Math 1204, 2205, 3205, 3207
Nova Scotia	Advanced Math 11, Advanced Math 12, Precalculus Math 12
Prince Edward Island	Math 421A, 521A, 521B, 621B, 611B

Every high school student in Atlantic Canada who intends to study calculus at university for the purpose of a degree in science should be able to demonstrate total comfort and facility with basic arithmetic, basic algebra, trigonometry, coordinate geometry and functions. University professors, and not just those of mathematics, routinely see errors like

1. $(x^2-1)y = x^2-1$ implies $y=0$;
2. $\sqrt{9} = \pm 3$;
3. $\sin 2x = 2\sin x$;
4. $\dfrac{\sin 6x}{x} = \sin 6$;
5. $\sqrt{x^2+y^2} = x+y$;
6. $\dfrac{1}{x+y} = \dfrac{1}{x} + \dfrac{1}{y}$;

and are overwhelmed by students' inability to

1. write $\frac{1}{2} + \frac{3}{5}$ as a fraction;
2. simplify $\dfrac{(a^3)^{-2}}{a}$;
3. simplify $\dfrac{\frac{1}{x}+2}{2-\frac{y}{3}}$;
4. compute $\dfrac{1}{x+1} + \dfrac{1}{x-1}$;
5. rationalize the denominator of $\dfrac{1+\sqrt{x}}{1+2\sqrt{x}}$.

These shortcomings clearly demonstrate a lack of understanding of concepts which were fundamental in their parents' schooldays and are still so today. They also explain why so many students have trouble with university level courses.

Across the region, placement/assessment/pre- tests for students wishing to enter a calculus course are becoming common. They are already in effect at Acadia, Memorial, the Universities of New Brunswick and Prince Edward Island, and at St. Mary's University, all of which require "satisfactory" performance for admission. Dalhousie has a diagnostic test for providing advice. The purpose of these tests is clear. They provide unequivocal evidence of the state of preparedness for university mathematics of each year's cohort and they make it easy to monitor trends in this state. Not only will universities without placement tests risk admitting poorly prepared students, but also, they will reduce the benefit which would otherwise be derived from a common database which universities and schools could use to improve mathematics education generally.

Recommendation 2. *Every university in Atlantic Canada should administer a placement test to all students wishing to study calculus for the purpose of a degree in science. Students who do not have a good chance of success in a calculus course should not be permitted to register.*

Since there is now a curriculum common to the region, it makes sense for universities to have common entrance standards.

Recommendation 3. *Universities in Atlantic Canada should share their current placement tests with each other and work towards the creation of a single test which would serve all.*

By "single test", we of course mean "multiple versions of the same test". We note that every student intending to take first year calculus at the University of New Brunswick receives a letter which includes a selection of problems typical of (or harder than) those which appear on the placement test. This seems to be a wise and helpful practice which others may wish to emulate.

University faculty have perceived a decrease in the skills and abilities of first year mathematics students for some years now, long before the current changes in curriculum. Will the new curriculum help better prepare students or will it make matters worse? Without hard evidence, it will be difficult to say.

Recommendation 4. *Every university in Atlantic Canada should track the performance of students who have experienced the full three years of the new curriculum and distribute the results to schools and universities throughout the region.*

At least one university in this country provides annual feedback to schools about the performance of their students in first year calculus.[2] While this action might prove contentious in some jurisdictions, we think the idea is excellent and that implementation by a majority

[2] See `http://www.math.ubc.ca/Schools/First_Year_Calculus_Results.htm`.

of the universities in Atlantic Canada acting upon a recommendation by APICS is most certainly possible. Remember that the goal is to have well prepared students and lower failure rates, whatever the curriculum.

Recommendation 5. *Universities in Atlantic Canada should provide annual feedback to school districts and high schools on the performance by their students in first year mathematics.*

Several years ago, working on behalf of the APICS Mathematics/Statistics Committee, Dr. Robert Dawson of St. Mary's University prepared a booklet, the intent of which was to show high school students what mathematical skills they should have before starting university calculus. The booklet, available online at `www.math.mun.ca/~apics/`, has undergone several revisions and continues, in our view, to be the best single source of advice to students and parents about university level calculus. It continues to be downloaded to sites around the world as many as 40 times a day.

Recommendation 6. *Departments of Mathematics, Faculties of Science and Student Advising Centres in the region should promote the APICS Calculus booklet through departmental and faculty web sites, posters and literature, and distribute hard copies, free of charge, to guidance counsellors and high school teachers of mathematics in their respective provinces.*

Many of the problems with school curricula have their genesis in and/or are exacerbated by the fact that teachers are not comfortable with the subject. No curriculum taught by anxious teachers will succeed; on the other hand, almost any curriculum taught by people who know and love the subject can work. We have heard reports of politics, prejudice and personality issues all too often providing the basis for teaching assignments.

The next recommendation is necessarily loose and we recognize that it cannot be enforced overnight; nonetheless, we feel it vital to stress that the number one priority in the assignment of teachers to mathematics courses should be qualifications.

Recommendation 7. *Departments of Education should endorse the principle that high school mathematics courses be taught by people who have qualifications indicating significant education in mathematics at the university level. Suitable qualifications (in order of preference) would be*

- *a major or honours degree in mathematics or closely related subject such as statistics, computing science, engineering, or physics;*
- *a minor in mathematics or a recognized certificate from a university programme in mathematics for high school teachers.*

Such qualifications should take priority over seniority in allocating high school mathematics teaching positions.

Coupled with this recommendation should be a commitment by mathematics departments in the region to encourage good students to become teachers, at every level in the school system.

For some period of time, teachers-in-training in the Atlantic region will be handicapped for a number of reasons. They will not have received their own mathematics education under the new curriculum. They will not have had the professional development which today's teachers have had. They may not be familiar with current technologies and other instructional supports. It is important that teacher education programmes recognize these realities and make special efforts to provide direction and support to novice teachers adapting to new methods.

Recommendation 8. *During the period when teachers-in-training have not had any direct experience with the new curriculum, teacher education programmes should make special efforts to provide additional workshops/training sessions to encourage familiarity with the new content, methodology and technologies.*

5 Everybody must get involved

Through the writing and promotion of this report, the APICS Mathematics/Statistics Committee hopes to increase public awareness of the problems associated with the transition from high school mathematics to university calculus. We have suggested ways in which deans and others in positions of authority can start to address the problem, but everybody involved in and affected by the education of our young people must participate as well.

5.1 Parents

We are astonished at the recent tendency of parents who, observing poor math performance by their children, hire a tutor and hope the problem will disappear. **Parents must get personally involved in their children's education and hold their children and teachers accountable.** Is your child being challenged? If your child fails a math test, ask yourself how much ownership of the material he or she took during the period leading up to the test. How much math homework did your child do on each of the seven nights prior to the test? If your child does well on a math test, ask the same question! "How much math homework does my child do night after night?" Mathematics cannot be crammed. Print off a copy of the APICS Calculus booklet[3] and take it to a parent-teacher night. Ask the teacher to show you which chapters in the math curriculum address the topics listed.

Contrary to what some people may believe, it is not the case that children should be able to complete all assigned work in school. Demand that your child be assigned homework beyond that completed in class. Demand that at least one sample of work be submitted and marked, with feedback, every week or so. Your son or daughter deserves this much

[3] `www.math.mun.ca/~apics/`

attention. And don't wait until Grade 12 to get involved. Every child should practice mathematics at every possible opportunity, especially through the completion of homework on a regular basis. If your child claims not to have math homework, contact the teacher and find out for yourself!

5.2 Teachers

Teachers face huge challenges. Classes are often large, student abilities are diverse, some classes are disrupted by behavioural problems, some students face problems at home that dwarf academic concerns. Many high school students have no intention of studying mathematics at university. Many students have no intention of attending university at all. We respect their decisions. While it should go without saying that an undergraduate degree is now just about the entrance standard for most jobs in this technologically based world, some of us question whether society puts too much pressure on teenagers to move **directly** from high school to university. Nevertheless, students who plan to take a university mathematics course should be prepared for it and, towards this end, they should expect their teachers to be prepared for the subjects they teach. Students of mathematics deserve to be taught by specialists with a sound knowledge of their subject. As is reinforced by comments in Appendix B, it is essential that inservice training sessions to help teachers understand and learn the new curriculum be put into place quickly. We also suggest the wisdom of welcoming university mathematics professors to these sessions and to the need for serious listening on both sides.

Creation and maintenance of a positive climate for teacher professional development, and for open communication and information exchange among all partners in mathematics education (parents, the post-secondary community, teachers), have never been more important. For a serious commentary on what teachers of mathematics should know, we refer the reader to a report by the U.S. Conference Board of the Mathematical Sciences[4] and to a discussion of this report which recently appeared in the *Notes of the Canadian Mathematical Society* **34** (March 2002), no 2.

5.3 Students

Finally, we encourage students themselves to assume more responsibility for their own education. The costs of education are well known, as is the demand for better education throughout the third world. In a country with so many advantages, it is criminal to let a day go by without taking full advantage of opportunities for education and self-development. Education is a serious business. Use every school minute to prepare for life's next challenge. Do your homework. Treat it seriously. If you have problems, see your teacher as soon as they start. Don't wait. Help is available. Use it!

[4] *Issues in Mathematical Education*, Volume 11, AMS & MAA 2001, ISBN 0-8218-2899-1, available at `http://www.cbmsweb.org/MET_Document/index.htm`

6 Acknowledgements

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The content of two previous reports on the new curriculum, by Barry Monson, Mark Taylor and Maureen Tingley, provided essential background material.

Maxim Burke (UPEI), Robert Dawson (St. Mary's),
Edgar G. Goodaire (Memorial), Paul Gosse (Prince of Wales Collegiate),
and Maureen Tingley (UNB)

on behalf of the
APICS Mathematics/Statistics Committee

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APPENDIX A

Courses and Syllabi

Newfoundland and Labrador

Program Level	*Level I*	*Level II*	*Level III*	*Course 4*
Advanced	1204	2205	3205	3207
Academic	1204	2204	3204	

Math 1204

- Data management: a statistics unit which include topics like lines of best fit which are part of some second year university courses[5]
- Networks and matrices: This includes network problems using graphs and digraphs, adjacency matrices and matrix multiplication, topics which appear in third year at Memorial University, for instance
- Patterns, relations, equations and predictions: students solve linear equations graphically and algebraically
- Modeling functional relationships: the concept of function
- How far? How tall? How steep? Right angle trigonometry
- The geometry of packaging: relationships between volume and surface area, a variety of shapes of containers being studied
- Linear programming: includes solution of systems of equations and inequalities involving two unknowns, a standard university business course topic

Math 2204/2205

- Equations in 3-space, algebra: 3-space, the solution of systems of equations using inverse matrices, standard university material
- Independent study unit: students explore a mathematical topic independently and make presentations to the class
- Trigonometry 1: periodic behaviour, the sine function

[5] We appreciate that the emphasis and level of treatment may be different.

- Trigonometry 2: the algebraic solution of trigonometric equations, trigonometric identities
- Statistics: using survey results to draw inferences about a population, confidence intervals, hypothesis testing, χ-square tests, all second year material at many universities
- Applications of trigonometry: the laws of sines and cosines, the area of a triangle

Mathematics 3204/3205

- Quadratics: arithmetic and geometric sequences, regression, the quadratic formula
- Rate of change: students discover how the shape of a graph indicates change and how the slope of the tangent relates to change
- Exponentials: including laws of exponents and the logarithm
- Circle geometry: transformational, euclidean and coordinate geometry, slopes of parallel and perpendicular lines, the equations of circles and ellipses
- Probability: counting principles, conditional probability, permutations and combinations, Pascal's triangle and the binomial expansion (second year material at Memorial)

Mathematics 3207

(Recommended for students who want direct entry to calculus at the post-secondary level)

- Sequences and series: the concept of limit, sigma notation, the area under a curve, the slope of a tangent, mathematical induction (first and second year university material)
- Complex numbers: rectangular and polar form, DeMoivre's Theorem
- Developing a function toolkit: composition of functions, inequalities, the derivative, graphs of rational functions, quadratic equations and their inverses
- Trigonometry: arc length, trigonometric identities (deeper study), inverse trigonometric functions

New Brunswick

Program Level	*Grade 10*	*Grade 11*	*Grade 12*	*Course 4*
Enriched Academic	Math 10	Math 111 (2 semesters)	Math 121	Math 120
Academic	Math 10	Math 112 (2 semesters)	Math 122	Math 120
Nonacademic	Math 10	Math 113 (2 semesters)		
Graduation	Math 10	One semester		

Math 10

The same as Math 1204 in Newfoundland.[6]

Math 111/112 (2 one-semester courses)

First Course (required for graduation)—Geometry and Applications in Mathematics ("Applications in Mathematics" in the nonacademic stream): Units 2 and 5 from Newfoundland 2204/2205 (independent study, statistics); units 4 and 5 from Newfoundland 3204/3205 (circle geometry, probability);

Second Course (not required for graduation)—Functions and Relations ("Patterns and Relations" in the nonacademic stream): Unit6 from Newfoundland 2204/2205 (sine and cosine laws); units 1, 2, 3 from Newfoundland 3204/3205 (quadratics, rate of change, exponentials)

There are two versions of the Math 10 and 11 courses, academic and nonacademic, with slightly different content. Many schools have traditionally also offered two levels of the academic stream though there is continual pressure (from the N.B. Department of Education) to drop the higher level, despite continual frantic appeals from parents who, of course, want the best for their children.

Math 121/122: Trigonometry and 3-space

Units 1, 3, 4 from Newfoundland 2204/2205 (equations in 3-space, trigonometry); unit 4 from Newfoundland 3207 (inverse trigonometric functions)

The three courses just described correspond to the two courses Math 2204/05 and 3204/05 in Newfoundland.

Math 120: Advanced Math with Calculus

Units 1, 2, 3 from Newfoundland 3207 (sequences and series, complex numbers, a function toolkit); a unit called "polynomial functions and derivatives".

[6] Here, and throughout the rest of this appendix, Newfoundland courses are treated as benchmarks by which those in other regions are sometimes compared. This decision simply reflects the author's home province.

Nova Scotia

Program Level	*Grade 10*	*Grade 11*	Grade 12	*Precalculus*
Advanced		Math 11	Math 12	Precalc Math 12
Academic	Math 10	Math 11	Math 12	
Graduation	Math Fnds 10	Math Fnds 11	Math Fnds 12	

Math 10

This is Newfoundland Math 1204. As in Newfoundland, it is offered at two levels, called "graduation" and "academic". The "graduation" course is called "Math Foundations 10." The "academic" course, Math 10, is higher level and appropriate for university bound students.

Math 11

This course is offered at three levels, graduation, academic and advanced. The Math 11 and Advanced Math 11 courses are roughly Newfoundland Math 2204/2205.

The Math Foundations 11 course sacrifices some trigonometry and statistics in order to introduce a unit on "consumer math."

- Making choices—Linear Programming: review linear characteristics; inequalities and feasible regions; describing and graphing constraints; connecting the feasible region and the solution
- Independent study: research and present new mathematics
- Decision making in consumer situations: income and deductions; budgets; financing and saving; transportation costs
- Statistics: surveys, sampling and inferences; collection, displaying and interpreting data; distributions and prediction; normal distribution
- Trigonometry: application; applying the law of sines and cosines

Math 12

As with Math 11, this course is also offered at three levels. The Math 12 and Advanced Math 12 courses are Newfoundland's Math 3204/3205.

The Math Foundations 12 course deletes the unit on rate of change in favour of one on patterning.

- Sequences (patterning): arithmetic, power and geometric sequences
- Quadratics: patterns and properties, exploring graphs, solving the quadratic in applications
- Exponential growth: patterns and properties, graphs, compound interest, some properties of exponents
- Circle geometry: properties and relationship theorems, distance and midpoint, informal proof, inductive and deductive thinking
- Probability: fundamental principle of counting, tree and area diagrams; simulations; distinguish between permutations and combinations; factorial notation; permutations and probability

Precalculus Math 12

The syllabus for this course varies somewhat from the Newfoundland precalculus course 3207.

- Sequences and series: recursion, arithmetic and geometric sequences and series; proofs by induction, limits; area under a curve
- Functions I: combinations and compositions; slopes, rate of change, limits, derivative; polynomial equations and functions; power rule; max/min problems
- Functions II: rational functions and asymptotes; irrational functions; absolute value function; continuity, limits and piecewise functions; modeling with exponential and logarithmic functions; the definitions and applications of e^x and $\ln x$
- Trigonometry: radian measure, the tangent function; reciprocal trig functions; combinations of trig functions; the general rotational matrix; trig identities; inverse trig relations
- Complex numbers: rectangular and polar forms; graphs; De Moivre's Theorem

Prince Edward Island

Program Level	*Grade 10*	*Grade 11*	*Grade 12*	*Course 4*
Advanced	421A	521A, 521B	621B	611B
Academic	421A	521A	621A	

Math 421A

Sequences and series; polynomials; relations and functions; coordinate geometry; trigonometry; data management

Math 521A

Systems of linear equations; quadratic functions; trigonometry; consumerism; matrices and networks

Math 521B

Radicals; reasoning, justification and proof; plane and coordinate geometry; linear equalities; rational expressions; equations, inequalities and developing a function toolkit

Math 621A

Transformations; exponents and logarithms; sequences and series; trigonometric functions; combinatorics and probability; statistics

Math 621B

Transformations; exponents and logarithms; sequences and series; trigonometric functions; combinatorics and probability; conics

Math 611B

This is roughly Newfoundland 3207.

Advanced trigonometry; complex numbers; functions; limits; derivatives and applications; integration and applications

APPENDIX B

Reflections on the New Curriculum

by
Paul Gosse[7]

1 History

The Atlantic Provinces Education Foundation (APEF) began as the Maritime Provinces Education Foundation (MPEF) in 1985. Newfoundland joined in 1995 to foster a uniform school mathematics standard across the region and to unify resources with the hope of reducing costs to the provinces.

APEF offices (located in Halifax) established a working group which then set responsibility for mathematics curriculum guide development. Newfoundland developed the intermediate guide, Nova Scotia the high school course guides and New Brunswick the primary/elementary guides.

As these guides were developed, publishers were invited to submit proposals for text and resource development to match these guides in their entirety. International Thomson Publishing was awarded this work at the high school level and struck author teams for two text series, "Mathematical Modelling" for university bound students and "Constructing Mathematics" for others. There is a teacher resource binder with each text. Irwin ("Interactions") and Pearson Education ("Minds on Math" and "Quest2000") produced the texts for the primary/elementary and junior high school streams.

2 The Pilot Process

In the past, the process for piloting new curricula involved soliciting texts which best met the needs of a particular course and pilot teachers tried to find a relatively polished resource which best fit syllabi.

The pilot process for APEF was dramatically different. Not only was the guide in **draft form** for most of the process, but so were the texts. In most cases, it could be argued, the pilot teachers were in fact rough copy editors, an experience which lasted up to three years for some teachers, and their students.

Repercussions of this new process on several fronts could have been anticipated. The first generation of graduates would likely not be representative of future students, as materials

[7]Mr. Gosse is head of the Mathematics Department at Prince of Wales Collegiate in St. John's, Newfoundland. He has taught first year mathematics courses, and primary/elementary and intermediate/secondary mathematics methods courses at Memorial University. He has led teacher inservice training sessions in Newfoundland on many occasions. As well, he has been actively involved providing feedback to and proofreading curriculum guides, and with inservice on course content and graphing calculator use in the new curriculum.

were dramatically refined. Teachers would likely struggle to complete courses as author teams and publishers refined time lines and approaches throughout the pilot process. In many cases, texts were not ready for general distribution until late summer, so preparation by teachers was impossible until they returned to school in September.

Combined with a dramatic shift in approach and a marked shift in emphasis (from more traditional algebra with significant practice to investigative processes and a statistics theme), the need to become deeply familiar with a substantial bundle of teachers' resource material and with very limited professional development time, it is not surprising that the first few years of implementation were fraught with teacher frustration.

3 Texts

The texts written for the new curriculum are not the sort to which most people are accustomed. Most of the material is expository, not example-driven. The texts provoke discovery by asking questions like "What do you notice?", "What is your conclusion?" Therefore, they do not seem review friendly nor are they particularly useful for last minute cramming. The answers to only certain exercises are in the back. The student generates notes, while the teacher attempts to summarize and distil the essentials from each "Investigation" and "Focus".[8] Practice sets are limited, though additional practice and some summary notes appear at the end of each chapter.

4 Content

This new path to calculus has been designed along a four-course route instead of the traditional three. Thus, topics like conic sections, rational expressions, exponentials and factoring, formerly spread over three courses in high school, are now stretched over four, often with depth of treatment and/or amount of practice altered. The content is also more modular and less spiral. A statistics theme occupies part of the first three courses. The fourth course (3207 in Newfoundland) is essentially pre-calculus with some calculus. One concern, under current course and unit alignment for example, is that a student sees nearly all precalculus trignometry in just the middle courses (2204/2205 in Newfoundland). For provinces with school-leaving examinations, a student may not treat these core calculus topics seriously if they know they will not appear on the examination.

5 Approach

Traditionally, mathematics texts were readily identifiable as such. Usually a section or topic would begin with a series of examples followed by a practice set. A teacher would follow and expand upon these examples and the student would complete assigned practice exercises to

[8] "Investigations" are student activities designed to develop a concept. "Foci" are teacher-lead topics not unlike traditional teaching.

encourage both understanding and efficiency. Some would argue that for many students, the ability to replay a well-rehearsed, context-sensitive algorithm or strategy, even with some efficiency, was no guarantee of real understanding.

APEF's courses take a different tack. Time once invested in teacher-lead note-taking is now invested in activities. Time once spent in gaining efficiency via practice sets is now spent answering a few germane questions designed to distil concepts from the activities. Student engagement in class time has never been more important. It is difficult to learn just by reading the text material; the student must also experience the material in order to generate a personalized set of learnings called "Notes", and **that** becomes their course or core study material, not the text itself.

Worthy of special note here is the pressure this puts on parents, tutors and friends. There is very little that looks familiar or affords a handhold into this material upon first glance. As one teacher recently put it, "it's as if the signposts are missing." When you look deeply enough and know what you are looking for, you can see where the development is going, but none of this is clear until you have carefully worked a unit.

An interesting effect on students, especially the mid-stream student, that I have noticed is that conceptual understanding in many areas seems to be improved. I see students who, in the past, would not have grasped how to generate an algebraic model for a sinusoidal wave now able to do so, and with understanding! In fact, today I commented that my 2204s could manage $y - 4 = 3\cos(2(x - \frac{\pi}{4}))$ while my 3201s[9] could not.

Efficiency seems to have been lost, however, and this is particularly evident in examination settings. My impression to date is that the new approach lends students a sense of comfort since they often do grasp the underlying concepts well. Unfortunately, this may sometimes translate into less urgency afforded homework and examination preparation. They may feel they know the material, but often have not had sufficient practice to automate skills. This is a programme deficit that may be possible to remedy with good teaching and perhaps second editions of the resources.

6 Technology

APEF's curriculum is intimately tied to the graphing calculator and manipulative technology. Literacy with the Texas Instruments TI-8* series of graphing calculator is necessary as are skills with manipulatives (such as algebra tiles), probability simulations, and Internet research. This is an additional expense on schools and students over time and has been incurred with limited professional development support.

[9] Math 3201 was the final advanced math course in the old Newfoundland curriculum

7 Teacher Preparation

The material is very difficult to prepare properly without a careful reading of the "Teacher Resource." While this is not necessarily a bad thing, it has an interesting side-effect. Parents, tutors, mentors, friends, advanced high school or university students wishing to help students are at a definite disadvantage if they don't have the Teacher Resource. Moreover, students who like to work independently or at a faster pace are disadvantaged by the new approach since the Teacher Resource is necessary support which they would not normally possess. This is not to say that teacher-made resources aren't valuable, but to observe that teachers may not have had to develop additional material to this extent before. This can be difficult where realignment of sequence for a topic, due to the discovery approach, can render many traditional worksheets unsuitable.

8 Teachers-In-Training

Most teachers-in-training in Atlantic Canada would have progressed through a much more traditional style of mathematics instruction in their secondary and post-secondary days. They would have no experience whatsoever in the methods required in the new APEF program.

This puts a tremendous onus on teacher training programmes, in particular upon methodology courses, to afford experience with the alternative approach while including an introduction to requisite technologies.

It is my experience so far that methodology courses lag behind current professional trends. Since current teachers-in-training will have missed initial teacher training inservice, and since a large portion of active teachers are poised to retire over the next five years, one wonders what the impact of failure to address this important need on the teaching of mathematics might be.

APPENDIX C

Implementation of the New Curriculum in Prince Edward Island

While Prince Edward Island is implementing the common curriculum, it made a decision in June 1998 to choose its own resources and develop its own process for implementing the curriculum, separately from the other provinces. The decision was the result of frustration with the process that had been followed by the APEF to that point. The PEI Department of Education's high school mathematics committee (which consists, roughly, of the Department of Education's mathematics consultant, a teacher from each high school and a representative from the University of Prince Edward Island) chose a textbook written for the western curriculum[10] and wrote resource materials for the teachers. The process in PEI has not generated the same controversy as has the process in the other provinces. In PEI, the High School teachers were directly involved in, and in control of, the process. The University of Prince Edward Island had the opportunity to provide regular input on the needs of its students.

[10] Manitoba, Saskatchewan, Alberta, British Columbia, the Northwest Territories, Yukon and Nunavut—see `http://www.learning.gov.ab.ca/news/1996nr/june96/nr-fourwpnr.asp` for more details on what these jurisdictions are doing