Saint Mary's University
DEPARTMENT OF MATHEMATICS
AND COMPUTING SCIENCE

Name: SOLUTIONS

Signature: ______

ID: ______

# Math 1251: Calculus for Life Sciences II

## Midterm Test

February 28, 2018

**Instructor:** J. Irving

**Instructions:**

- *No electronic devices, or aids of any kind, are permitted.*
- *There are 4 pages plus this cover page. Check that your test paper is complete.*
- *There are a total of 60 points. The value of each question is indicated in the margin.*
- *Answer in the spaces provided, using backs of pages for additional space if necessary.*
- *Show all your work. Insufficient justification will result in a loss of points.*

| Page | Maximum | Your Score |
|---|---|---|
| 1 | 19 | |
| 2 | 13 | |
| 3 | 12 | |
| 4 | 16 | |
| **Total** | **60** | |

1. Compute:

[7] (a) $\displaystyle\int \left(1 - \frac{x}{2} + \frac{3}{x} - 4x^4 + \frac{5}{\sqrt[3]{x}} - e^{6x} + \sin 7x\right) dx$

$= x - \frac{1}{4}x^2 + 3\ln|x| - \frac{4}{5}x^5 + \frac{15}{2}x^{2/3} - \frac{1}{6}e^{6x} - \frac{1}{7}\cos 7x + C$

[4] (b) $\displaystyle\int_0^1 t^2(1 - \sqrt{t})\, dt$

$= \int_0^1 (t^2 - t^{5/2})\, dt$

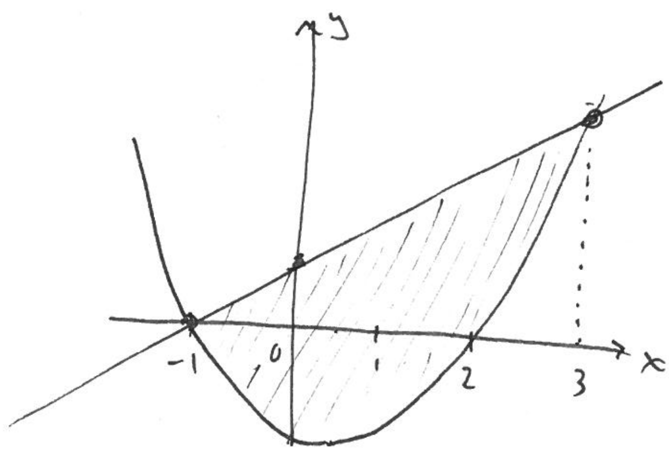
$= \frac{1}{3}t^3 - \frac{2}{7}t^{7/2}\Big]_0^1$

$= \frac{1}{3} - \frac{2}{7} = \boxed{\frac{1}{21}}$

[8] 2. Find the area of the region bounded between the curves $y = x + 1$ and $y = x^2 - x - 2$. Your solution should include an appropriately labelled sketch of the region.

$y = x^2 - x - 2 = (x+1)(x-2)$

Parabola with roots $-1, 2$




Solve $x + 1 = x^2 - x - 2$

$\Rightarrow 0 = x^2 - 2x - 3$

$\Rightarrow 0 = (x-3)(x+1)$

$\Rightarrow x = -1$ or $x = 3$

Then Area $= \int_{-1}^3 \left((x+1) - (x^2 - x - 2)\right) dx$

$= \int_{-1}^3 (2x + 3 - x^2)\, dx$

$= x^2 + 3x - \frac{1}{3}x^3\Big]_{-1}^3$

$= (9 + 9 - 9) - (1 - 3 + \frac{1}{3})$

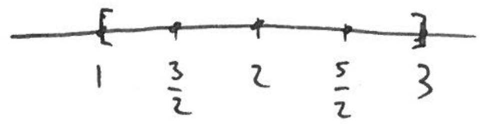
$= 11 - \frac{1}{3}$

$= \boxed{\frac{32}{3}}$

[4] 3. Approximate $\int_1^3 \sqrt{1+x}\,dx$ by a Riemann sum with $N = 4$ subintervals, using right-endpoints.

**Do not evaluate the sum!**

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$




$$\int_1^3 \sqrt{1+x}\,dx \approx \left(\sqrt{1+\tfrac{3}{2}} + \sqrt{1+2} + \sqrt{1+\tfrac{5}{2}} + \sqrt{1+3}\right)\cdot\frac{1}{2}$$

$$= \left(\sqrt{\tfrac{5}{2}} + \sqrt{3} + \sqrt{\tfrac{7}{2}} + 2\right)\cdot\frac{1}{2}$$

4. Give expressions in terms of definite integrals for the following.

**Do not evaluate your integrals!**

[3] (a) The distance travelled over the first 10 seconds by a particle moving in a straight line whose velocity after $t$ seconds is $1 - e^{-t}$ m/s.

$$\boxed{\int_0^{10} (1 - e^{-t})\,dt}$$

[3] (b) The volume of the solid obtained by revolving the portion of the curve $y = \ln x$ between $x = 1$ and $x = e$ around the $x$-axis.

$$\boxed{\pi \int_1^e (\ln x)^2\,dx}$$

[3] (c) The average value of the function $f(x) = \sqrt{1+x^2}$ over the interval $[1, 4]$.

$$\boxed{\frac{1}{3}\int_1^4 \sqrt{1+x^2}\,dx}$$

[4] 5. Compute $\dfrac{\partial}{\partial x} xy^2 \sin(yx^3)$.

$$y^2 \sin(yx^3) + xy^2 \cos(yx^3) \cdot 3x^2y$$

$$= y^2 \sin(yx^3) + 3x^3y^3 \cos(yx^3)$$

[8] 6. Find all critical points of $f(x,y) = 10 - x^2 + 2xy - 3y^2 - 4x + 16y$ and use the second derivative test to classify them as maxima, minima, or neither. **Clearly explain your reasoning.**

$$\frac{\partial f}{\partial x} = -2x + 2y - 4, \quad \frac{\partial f}{\partial y} = 2x - 6y + 16$$

Set $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$: $\begin{cases} -2x + 2y = 4 \\ 2x - 6y = -16 \end{cases} \Rightarrow -4y = -12$

$\Rightarrow y = 3$

$\Rightarrow x = 1$

So $(x,y) = (1,3)$ is the only critical pt.

Now: $\dfrac{\partial^2 f}{\partial x^2} = -2$, $\dfrac{\partial^2 f}{\partial y^2} = -6$, $\dfrac{\partial^2 f}{\partial x \partial y} = 2$

$$\Rightarrow D(x,y) = (-2)(-6) - (2)^2$$
$$= 8$$

Since $D(1,3) = 8 > 0$ and $\frac{\partial^2 f}{\partial x^2}(1,3) = -2 < 0$, conclude that $(1,3)$ is a local maximum.

7. Compute:

[5] (a) $\displaystyle\int_0^1 \frac{x^3}{(1+3x^4)^2}\,dx$

Let $u = 1+3x^4$, so $du = 12x^3\,dx$

and $x=0 \Rightarrow u=1$
$x=1 \Rightarrow u=4$

$$= \int_1^4 \frac{1}{u^2}\cdot\frac{1}{12}\,du$$

$$= -\frac{1}{12}\cdot\frac{1}{u}\Big]_1^4$$

$$= -\frac{1}{12}\left(\frac{1}{4}-1\right) = \boxed{\frac{1}{16}}$$

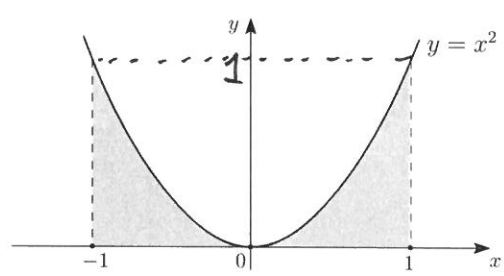
[5] (b) $\displaystyle\int xe^{2x}\,dx$

Let $u = x$, $dv = e^{2x}\,dx$
$du = dx$, $v = \frac{1}{2}e^{2x}$

Then $\displaystyle\int xe^{2x}\,dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$

$$= \boxed{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C}$$

[6] 8. The region $R$ of the $(x,y)$-plane is shaded in the sketch below. Find the net volume of the solid over $R$ that is bounded above by $f(x,y) = 1-2x^2y$.




$$\text{Volume} = \int_{-1}^{1}\int_0^{x^2} (1-2x^2y)\,dy\,dx$$

$$= \int_{-1}^{1} \left(y - x^2y^2\right)\Big|_0^{x^2}\,dx$$

$$= \int_{-1}^{1} \left(x^2 - x^2(x^2)^2\right)dx$$

$$= \int_{-1}^{1} (x^2-x^6)\,dx$$

$$= \frac{1}{3}x^3 - \frac{1}{7}x^7\Big]_{-1}^{1}$$

$$= \left(\frac{1}{3}-\frac{1}{7}\right) - \left(-\frac{1}{3}+\frac{1}{7}\right)$$

$$= \boxed{\frac{8}{21}}$$

OOPS: We've actually calculated "net volume" here, since the function $1-2x^2y$ can be negative over $R$. (I intended $f(x,y) = 1+2x^2y$.)

If you're curious, the true volume here is

$$\frac{32-29\sqrt[4]{2}}{21}$$

See me if you want to know why.