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Math 1215: Quiz #5 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. Let $f(x, y) = 2x^2 + 5xy + 5y^2 - 2x - y + 3$.

   (a) Find all possible points (x, y) at which f could have a local extremum.

6

$$\frac{\partial f}{\partial x} = 4x + 5y - 2, \quad \frac{\partial f}{\partial y} = 5x + 10y - 1$$

So $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \iff$ ① $4x + 5y = 2$, ② $5x + 10y = 1$ $\Rightarrow -3x = -3$ (② - 2×①)

$\Rightarrow x = 1$

$\Rightarrow y = \frac{-2}{5}$

So $(1, -\frac{2}{5})$ is the only critical point.

   (b) Apply the second-derivative test to determine whether the points found in (a) are local maxima, local minima, or neither.

4

$$\frac{\partial^2 f}{\partial x^2} = 4, \quad \frac{\partial^2 f}{\partial y^2} = 10, \quad \frac{\partial^2 f}{\partial y \partial x} = 5$$

So $D(x, y) = (4)(10) - 5^2 = 15$

Since $D(1, -\frac{2}{5}) = 15 > 0$ and $\frac{\partial^2 f}{\partial x^2}(1, -\frac{2}{5}) = 4 > 0$,

we know $(1, -\frac{2}{5})$ is a local minimum.

v5.2

$\frac{}{10}$

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Name: SOLUTIONS	A#:	Section:

1. Let $f(x, y) = 1 - 3x^2 + 6xy + y^2 - 5x - 3y$.

   (a) Find all possible points (x, y) at which f could have a local extremum.

$$\frac{\partial f}{\partial x} = -6x + 6y - 5 \qquad \frac{\partial f}{\partial y} = 6x + 2y - 3$$

So $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \iff \left.\begin{array}{l} -6x + 6y = 5 \\ 6x + 2y = 3 \end{array}\right\} \Rightarrow 8y = 8$

$\Rightarrow y = 1$

$\Rightarrow x = \frac{1}{6}$

(6)

$\therefore$ $(\frac{1}{6}, 1)$ is the only critical point.

   (b) Apply the second-derivative test to determine whether the points found in (a) are local maxima, local minima, or neither.

$$\frac{\partial^2 f}{\partial x^2} = -6, \quad \frac{\partial^2 f}{\partial y^2} = 2, \quad \frac{\partial^2 f}{\partial y \partial x} = 6$$

(4)

So $D(x, y) = (-6)(2) - 6^2 = -48$

Since $D(\frac{1}{6}, 1) = -48 < 0$, we conclude that $(\frac{1}{6}, 1)$ is a saddle point

(i.e neither max nor min)

v5.1