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Math 1251: Quiz #8 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. Determine whether the improper integral $\int_0^\infty \frac{dx}{(2+x)^3}$ converges or diverges.

$$\int_0^\infty \frac{dx}{(2+x)^3} = \lim_{b\to\infty} \int_0^b (2+x)^{-3}\,dx$$

$$= \lim_{b\to\infty} -\tfrac{1}{2}(2+x)^{-2}\Big|_0^b$$

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$$= \lim_{b\to\infty} \left(-\tfrac{1}{2}\cdot \frac{1}{(2+b)^2}^{\,0} + \tfrac{1}{8} \right)$$

$$= \boxed{\tfrac{1}{8}}$$

So it converges to 1/8.

2. Find the unique function $y(t)$ that satisfies $y' = te^{-y}$ and $y(0) = 1$.

$$\frac{dy}{dt} = te^{-y} \Rightarrow e^y\,dy = t\,dt$$

$$\Rightarrow \int e^y\,dy = \int t\,dt$$

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$$\Rightarrow e^y = \tfrac{1}{2}t^2 + C$$

$$\Rightarrow y = \ln\left(\tfrac{1}{2}t^2 + C\right)$$

Then $y(0)=1 \Rightarrow 1 = \ln(C) \Rightarrow C = e$

So $\boxed{y = \ln\left(\tfrac{1}{2}t^2 + e\right)}$

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