Math 1251: Worksheet #1 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. $\int \left(2x^5 - \frac{3}{4x^2} + \frac{e}{5}\right)\, dx$

$= 2\int x^5\, dx - \frac{3}{4}\int x^{-2}\, dx + \int \frac{e}{5}\, dx$

$= 2\cdot\frac{1}{6}x^6 - \frac{3}{4}(-x^{-1}) + \frac{e}{5}x + C$

$= \boxed{\frac{1}{3}x^6 + \frac{3}{4x} + \frac{ex}{5} + C}$

2. $\int e^x(1 + e^{4x})\, dx$

$= \int (e^x + e^{5x})\, dx$

$= e^x + \frac{1}{5}e^{5x} + C$

3. $\int \frac{\sin \pi x}{2}\, dx$

$= \frac{1}{2}\int \sin(\pi x)\, dx$

$= \frac{1}{2}\left(-\frac{1}{\pi}\cos \pi x\right) + C$

$= \boxed{-\frac{1}{2\pi}\cos \pi x + C}$

4. $\int \frac{4-x}{\sqrt[3]{x}}\, dx$

$= \int \left(4x^{-1/3} - x^{2/3}\right) dx$

$= 4\cdot\frac{3}{2}x^{2/3} - \frac{3}{5}x^{5/3} + C$

$= \boxed{6x^{2/3} - \frac{3}{5}x^{5/3} + C}$

5. Find all k such that $\int \frac{dx}{1-x^2} = k \ln\left(\frac{x+1}{x-1}\right) + C$ [*Hint:* First use logarithm rules!]

Differentiate the RHS:

$$\frac{d}{dx}\left(k \ln \frac{x+1}{x-1} + C\right) = \frac{d}{dx}\left(k \ln(x+1) - k\ln(x-1) + C\right)$$

$$= \frac{k}{x+1} - \frac{k}{x-1}$$

$$= \frac{k(x-1) - k(x+1)}{(x+1)(x-1)}$$

$$= \frac{-2k}{x^2-1}$$

This equals $\frac{1}{1-x^2}$ when $-2k = -1$, i.e. $\boxed{k = \frac{1}{2}}$

6. The function $y(x)$ satisfies $\frac{d^2y}{dx^2} = 5x^{2/3} - 1$ and $y(0) = -1$, $y(1) = 0$. Find $y(x)$.

Antidifferentiate to get $\frac{dy}{dx} = \int (5x^{2/3} - 1)\,dx$

$$= 3x^{5/3} - x + C$$

Do it again to get $y = \int (3x^{5/3} - x + C)\,dx$

$$= \frac{9}{8}x^{8/3} - \frac{1}{2}x^2 + Cx + D.$$

Since $y(0) = -1$, get $D = -1$

Then $y(1) = 0 \Rightarrow 0 = \frac{9}{8} - \frac{1}{2} + C - 1 \Rightarrow C = \frac{3}{8}$

So $\boxed{y(x) = \frac{9}{8}x^{8/3} - \frac{1}{2}x^2 + \frac{3}{8}x - 1}$