Math 1251: Worksheet #2 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. $\int_{-1}^{1} \frac{3}{e^{2t}}\, dt$

$= 3\int_{-1}^{1} e^{-2t}\, dt$

$= 3\left(-\frac{1}{2}e^{-2t}\right)\Big|_{-1}^{1} = -\frac{3}{2}\left(e^{-2}-e^{2}\right)$

$= \boxed{\frac{3}{2}\left(e^{2}-\frac{1}{e^{2}}\right)}$

2. $\int_{0}^{1} (1-x^2)^2\, dx$

$= \int_0^1 (1-2x^2+x^4)\, dx$

$= \left(x-\frac{2}{3}x^3+\frac{1}{5}x^5\right)\Big|_0^1 = \left(1-\frac{2}{3}+\frac{1}{5}\right)-0 = \boxed{\frac{8}{15}}$

3. Given that $\int_{-1}^{2} f(x)\, dx = 3$ and $\int_{-1}^{6} f(x)\, dx = 5$, find $\int_{2}^{6} f(x)\, dx$.

$\int_2^6 f(x)\, dx = \int_{-1}^{6} f(x)\, dx - \int_{-1}^{2} f(x)\, dx$

$= 5-3$

$= \boxed{2}$

4. Find all real numbers $b > 0$ such that the area under the graph of $y = \frac{x^2+1}{2}$ over the interval $0 \le x \le b$ is equal to b..

The area is $\int_0^b \frac{x^2+1}{2}\, dx = \frac{1}{2}\int_0^b (x^2+1)\, dx$

$= \frac{1}{2}\left(\frac{1}{3}x^3+x\right)\Big|_0^b$

$= \frac{1}{2}\left(\frac{b^3}{3}+b\right)$

Solve $b = \frac{1}{2}\left(\frac{b^3}{3}+b\right) \Rightarrow b^3-3b=0$

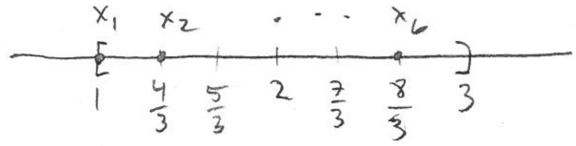
$\Rightarrow b(b^2-3)=0$

Since $b>0$, get $\boxed{b=\sqrt{3}}$

v2.1

5. (a) Approximate the area under the graph of $y = 1/x^2$ between $x = 1$ and $x = 3$ by a Riemann sum, with $N = 6$ subintervals and using left endpoints. You can leave your answer as a sum of fractions.

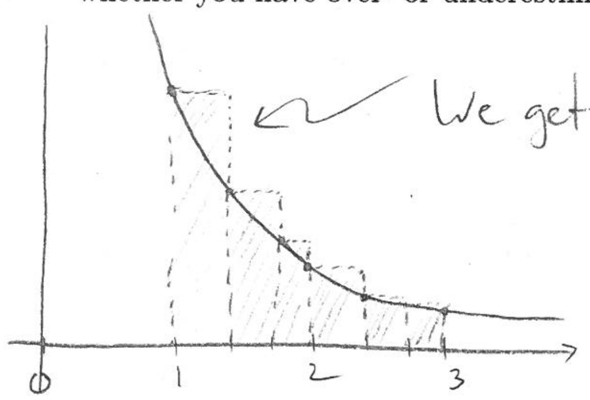
$\Delta x = \frac{3-1}{6} = \frac{1}{3}$



$$\text{Area} \approx \left(\frac{1}{1^2} + \frac{1}{(4/3)^2} + \frac{1}{(5/3)^2} + \frac{1}{2^2} + \frac{1}{(7/3)^2} + \frac{1}{(8/3)^2}\right) \cdot \frac{1}{3}$$

$$= 3\left(\frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \frac{1}{8^2}\right)$$

(b) Illustrate your approximation in (a) with a sketch. Use the sketch to determine whether you have over- or underestimated the area.



We get an overestimate of the true area.

(c) Find the true value of the area described in (a).

$$\text{Area} = \int_1^3 \frac{dx}{x^2} = -\frac{1}{x}\Big|_1^3$$

$$= -\frac{1}{3} - (-1)$$

$$= \boxed{\frac{2}{3}}$$

6. A water tank is being drained. After t minutes of draining, the water level in the tank is decreasing at a rate of $(8 - t)^{-2/3}$ metres per minute. How much does the water level decrease in the first 7 minutes?

$$\text{Decrease} = \int_0^7 (8-t)^{-2/3}\, dt$$

$$= -3(8-t)^{1/3}\Big|_0^7$$

$$= -3\left(1^{1/3} - 8^{1/3}\right)$$

$$= (-3)(1-2)$$

$$= \boxed{3} \text{ metres.}$$

v2.1